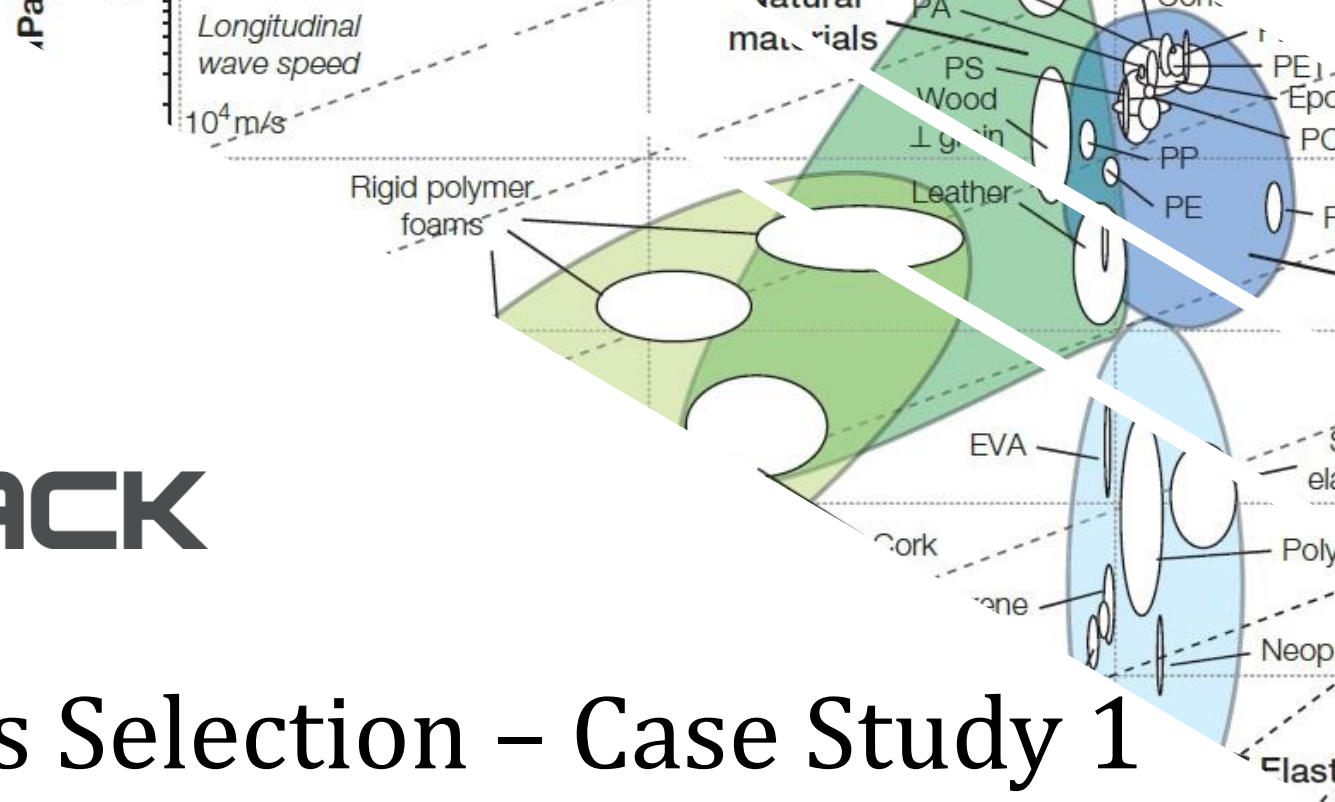




GRANTA  
**CES**  
**EDUPACK**



# Materials Selection – Case Study 1

## Bases and Mechanical Properties

Professors:

Anne Mertens and Davide Ruffoni

Assistant:

Tommaso Maurizi Enrici



# Mechanical Properties Case Studies

- *Case Study 1: The Lightest STIFF Beam*
- *Case Study 2: The Lightest STIFF Tie-Rod*
- *Case Study 3: The Lightest STIFF Panel*
- *Case Study 4: Materials for Oars*
- *Case Study 5: Materials for CHEAP and Slender Oars*
- *Case Study 6: The Lightest STRONG Tie-Rod*
- *Case Study 7: The Lightest STRONG Beam*
- *Case Study 8: The Lightest STRONG Panel*
- *Case Study 9: Materials for Constructions*
- *Case Study 10: Materials for Small Springs*
- *Case Study 11: Materials for Light Springs*
- *Case Study 12: Materials for Car Body*

CES 2009

CES 2016



# Materials selection

- **Mechanical properties:** tensile test, fatigue, hardness, toughness, creep...
- **Physical properties:** density, conductivity, coefficient of thermal expansion
- **Chemical properties :** corrosion
- **Microscopic characteristics:** anisotropy of properties, hardening, microstructure, grain size, segregation, inclusions...

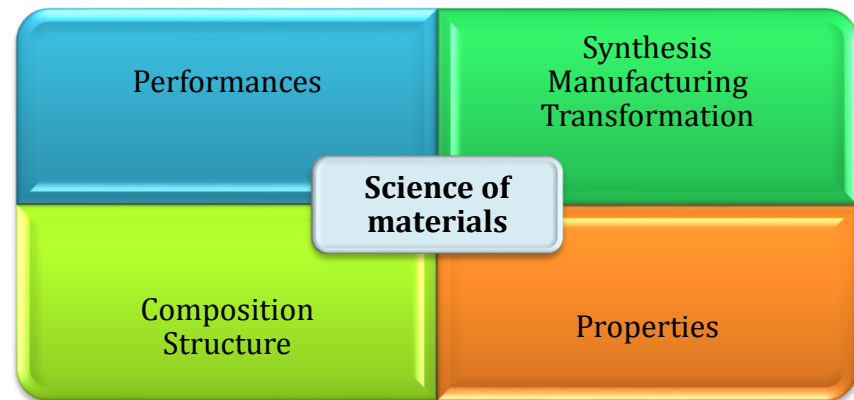


# Materials selection

- **Process linked aspects:** formability, machinability, weldability, stampability
- **Aesthetic aspects:** colour and surface roughness

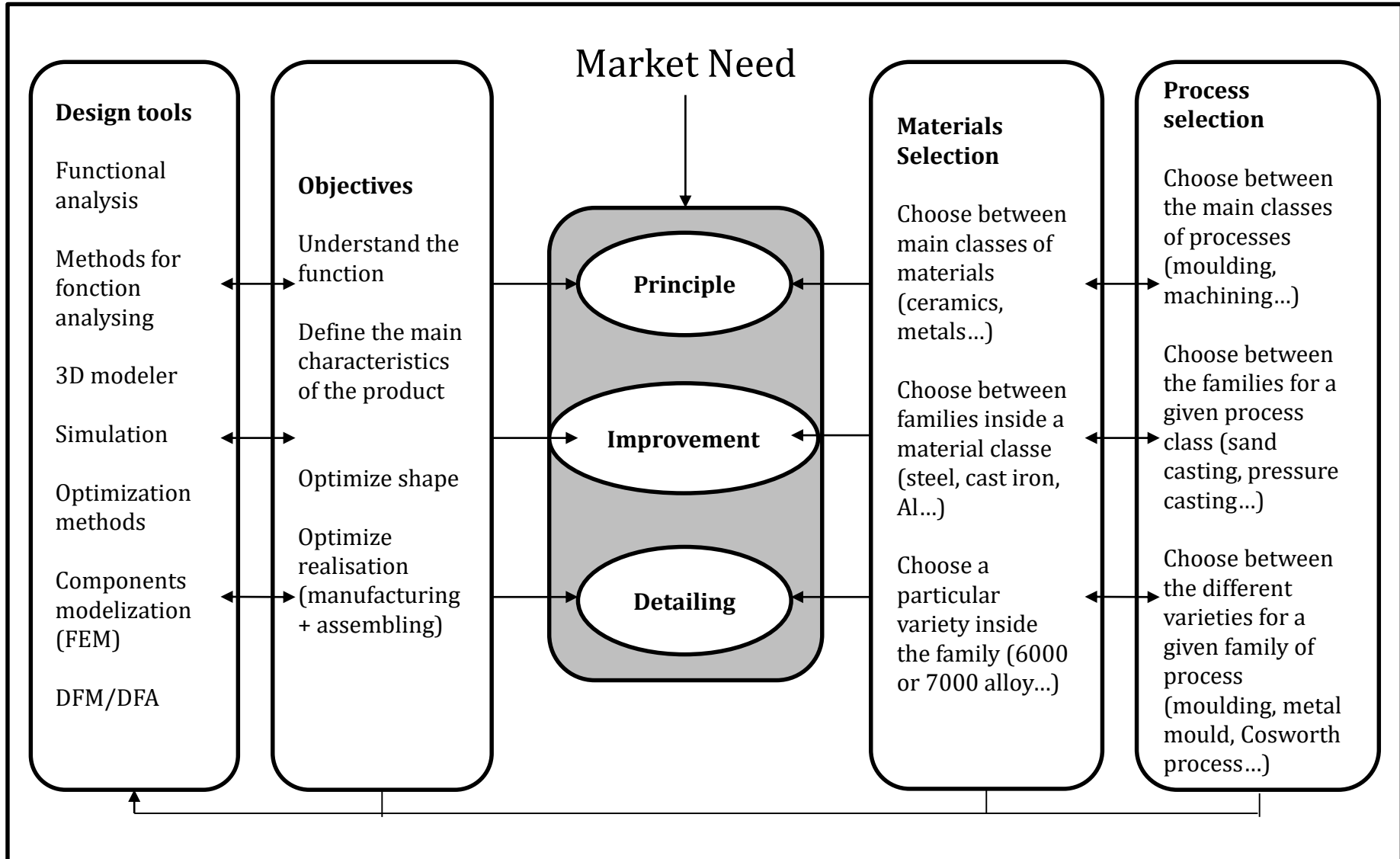
Notice: surface properties  $\neq$  volume properties

4 poles for  
engineering and  
material science





# Design steps





# Forecasts

**Evolution of materials** is challenged by:

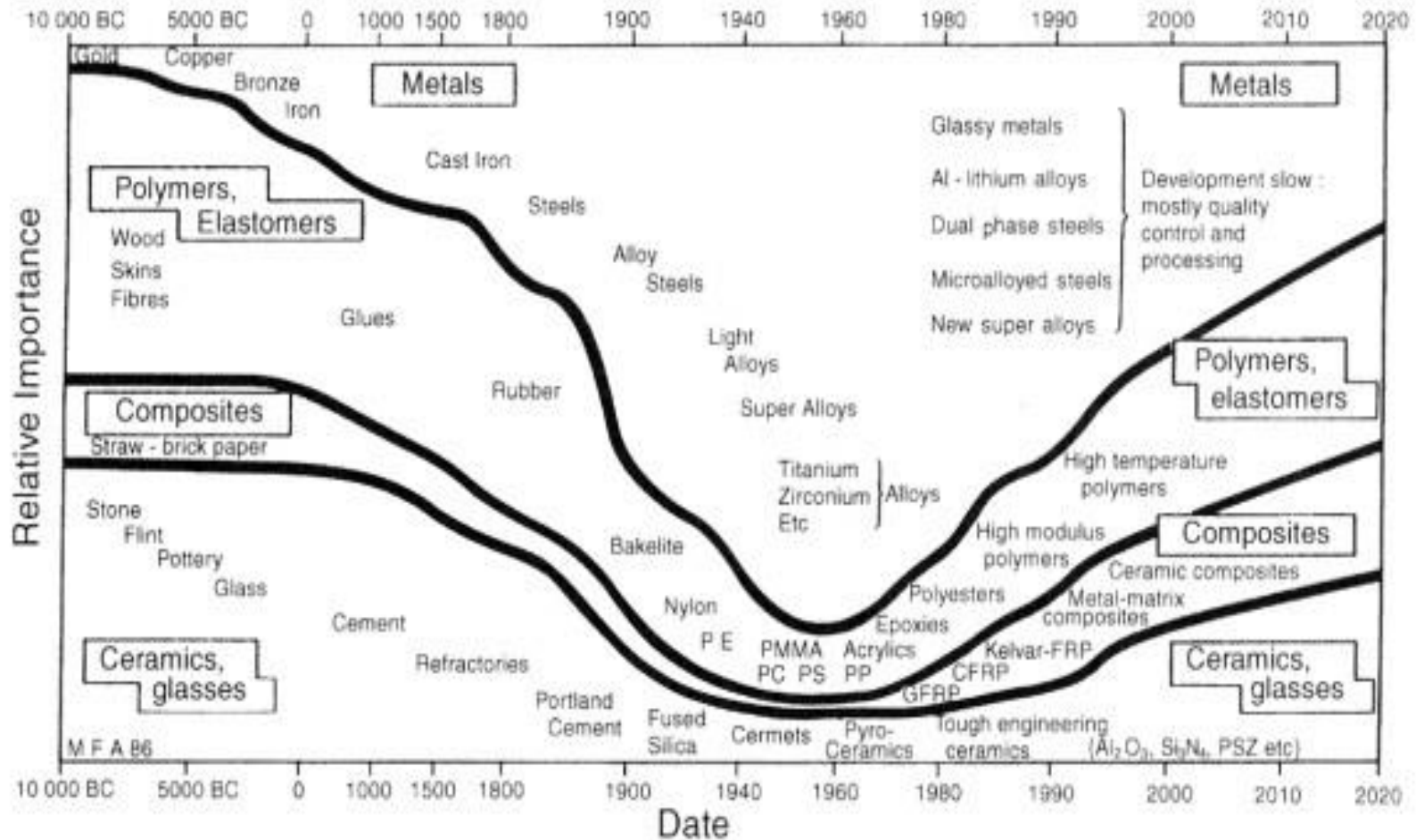
- ↗ mechanical properties
- ↗ physical and chemical properties
- ↘ environmental problems (manufacturing)
- ↘ materials resource

Key Domains : energy (nuclear, solar cells, ...)

transport



# A bit of History





# A bit of History



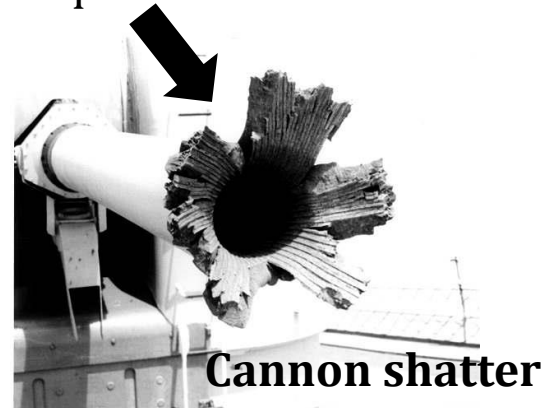
1850s, time of the Crimean War

Napoleon III

French military engineers had found they could control the trajectory applying a rifling or “spinning” in the barrels of guns (cannon)



The spiraling motion added extra stresses  
Consequence?



Need a higher-strength material  
→ **Steel**





# A bit of History

1946, University of Pennsylvania  
Moore School of Electrical  
Engineering



Electronic Numerical  
Integrator Analyser and  
Computer (**Eniac**) by **John  
Mauchly and J. Presper  
Eckert**



1947,  
Discovery of the Transistor  
(Semiconductors)

Built from materials such as  
**Silicon and Germanium** which  
can either behave as an electrical  
insulator or conductor

**The first general-purpose  
electronic computer**

17468 thermionic valves  
70,000 resistors

....  
Covered 167 square metres of floor space  
Weighed 30 tonnes  
Consumed 160 kW of electricity



Companies spent tens of  
billion of dollars to squeeze  
more circuits on to a small  
'chip' of material

2010, an Intel X3370 microprocessor – 820 million transistors  
**Your computer could handle 3 billion instructions /s  
600000 more than Eniac**



# A bit of History

2012, low cost airlines company

Change the material of a small pivot (46) for each seat

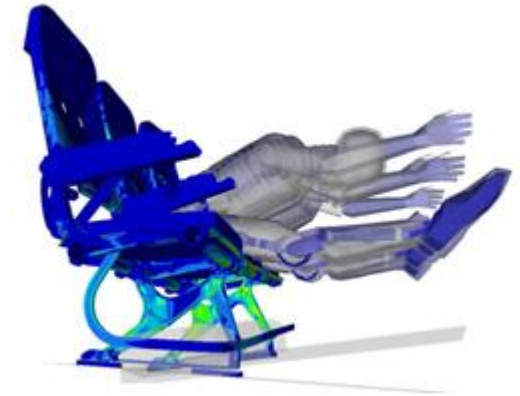
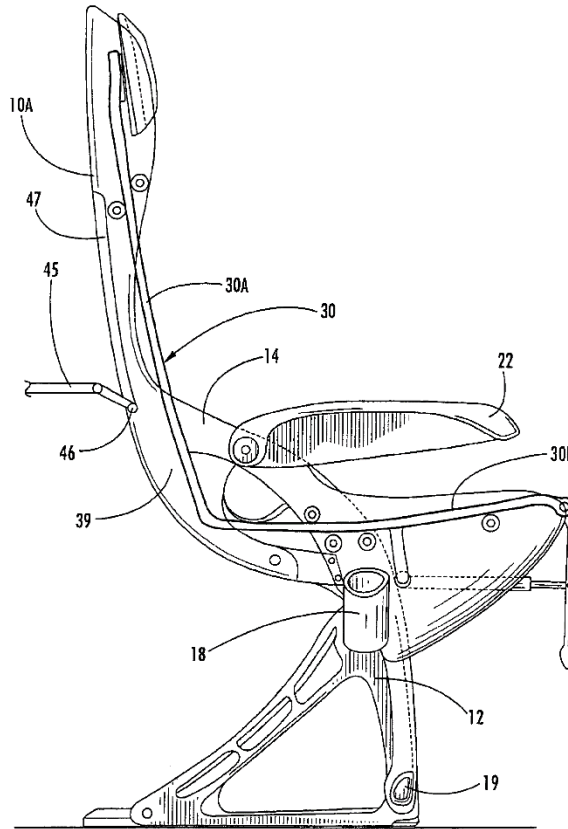


FIG. 2.

In the air transports  
Weight = Costs

Aluminum → PE+ Glass fibers Composite



10,000,000 dollars saved each year

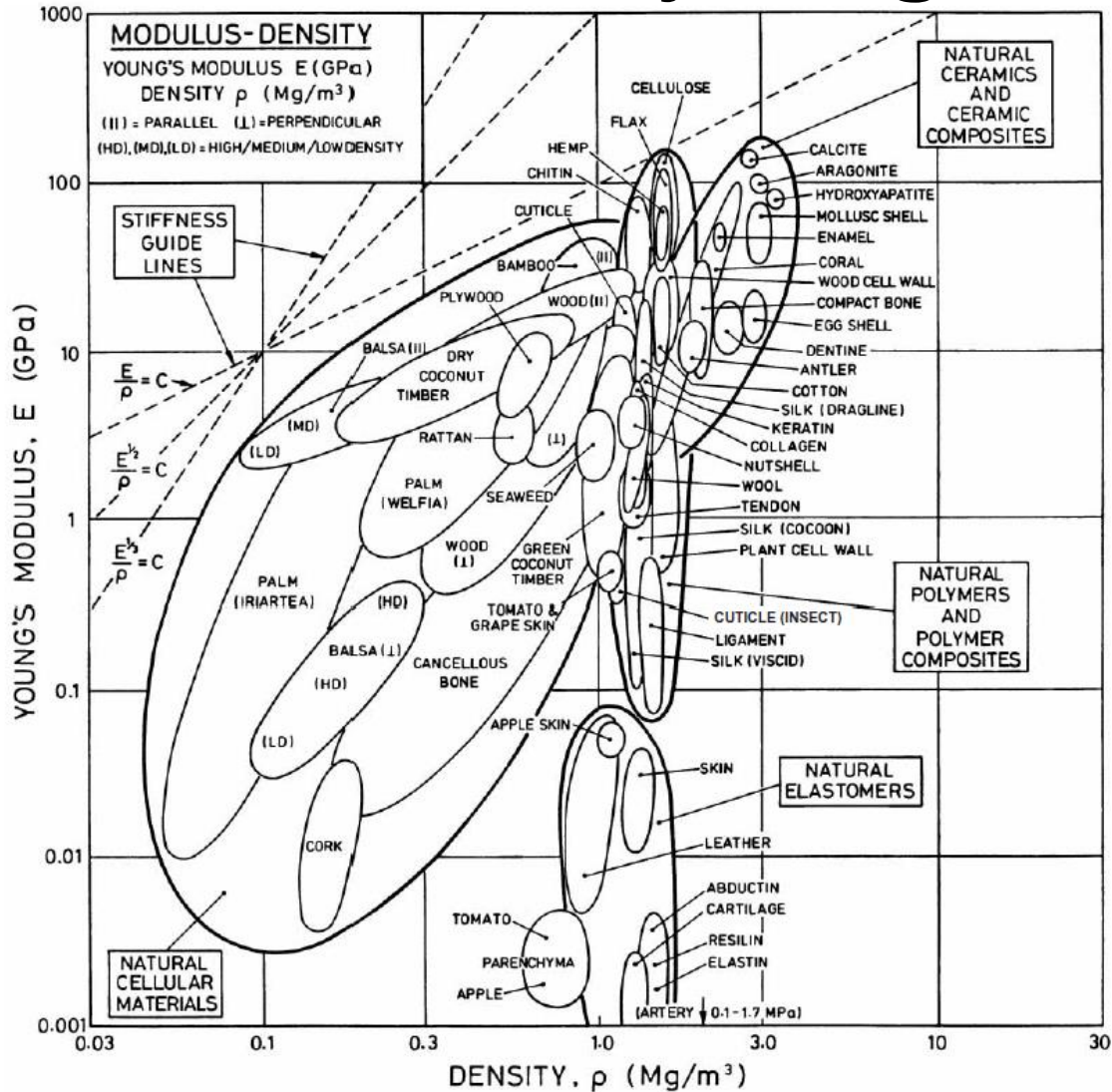


# Mike Ashby from University of Cambridge





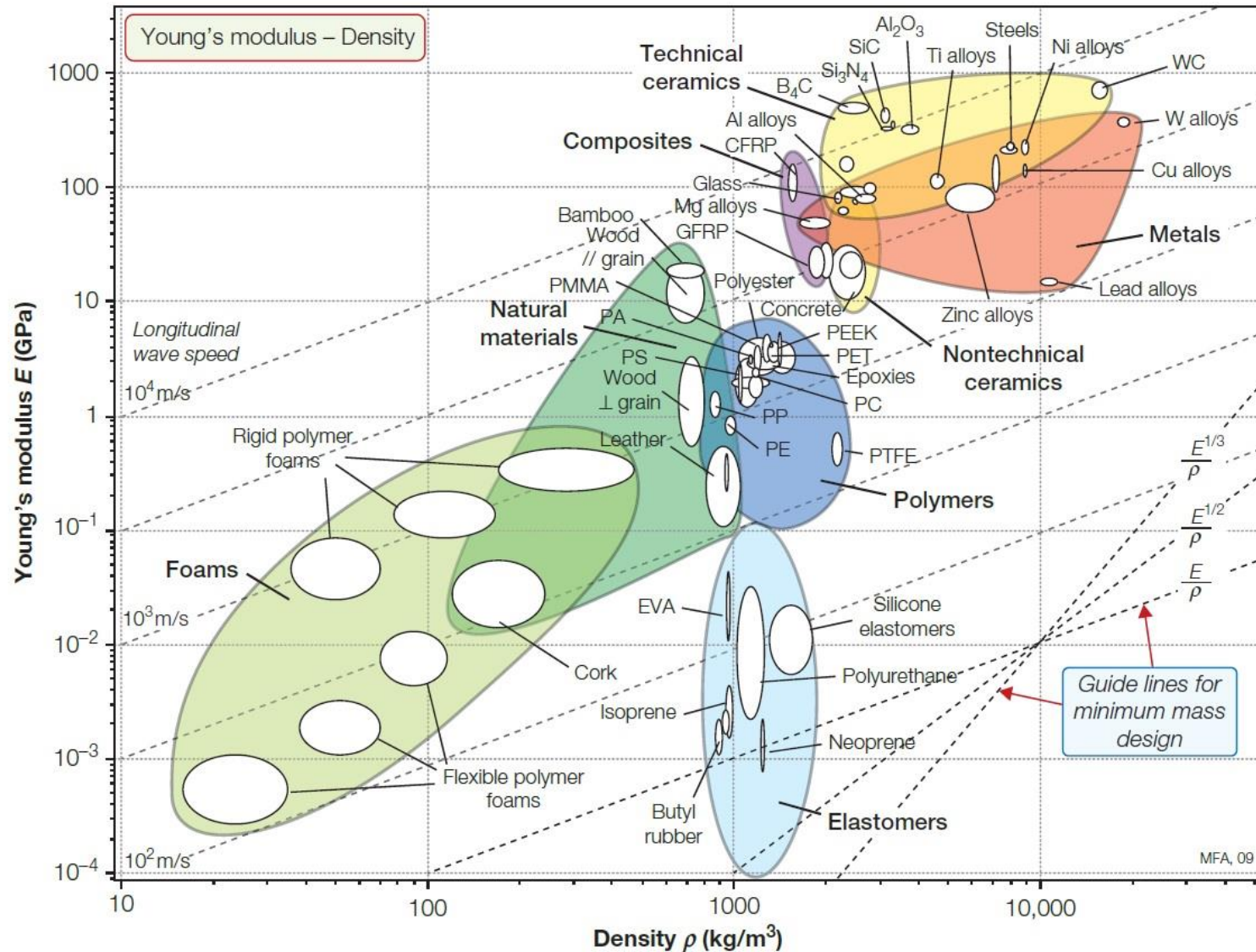
# Ashby Diagrams



[The mechanical efficiency of natural materials, Mike Ashby, 2003]

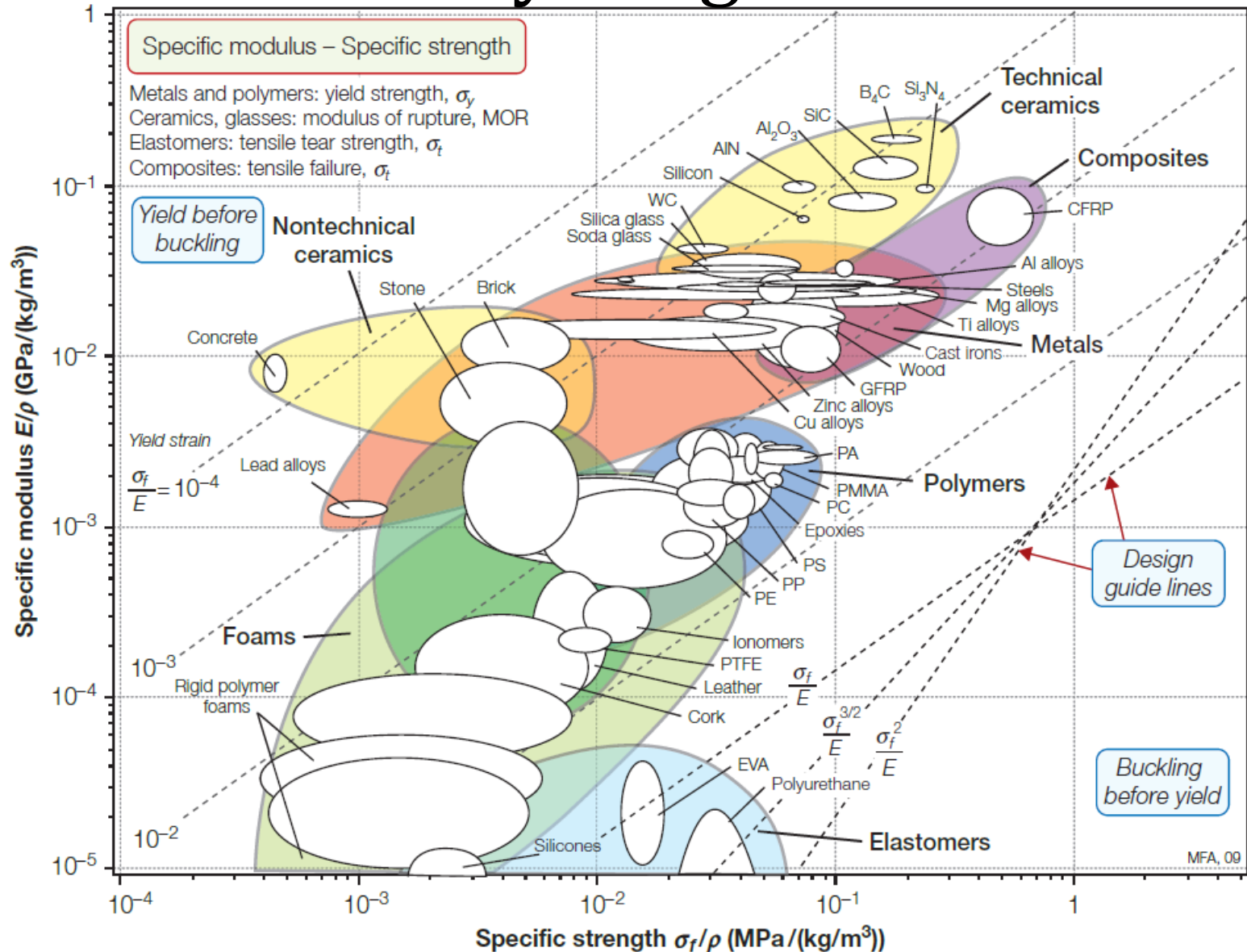


# Ashby Diagrams



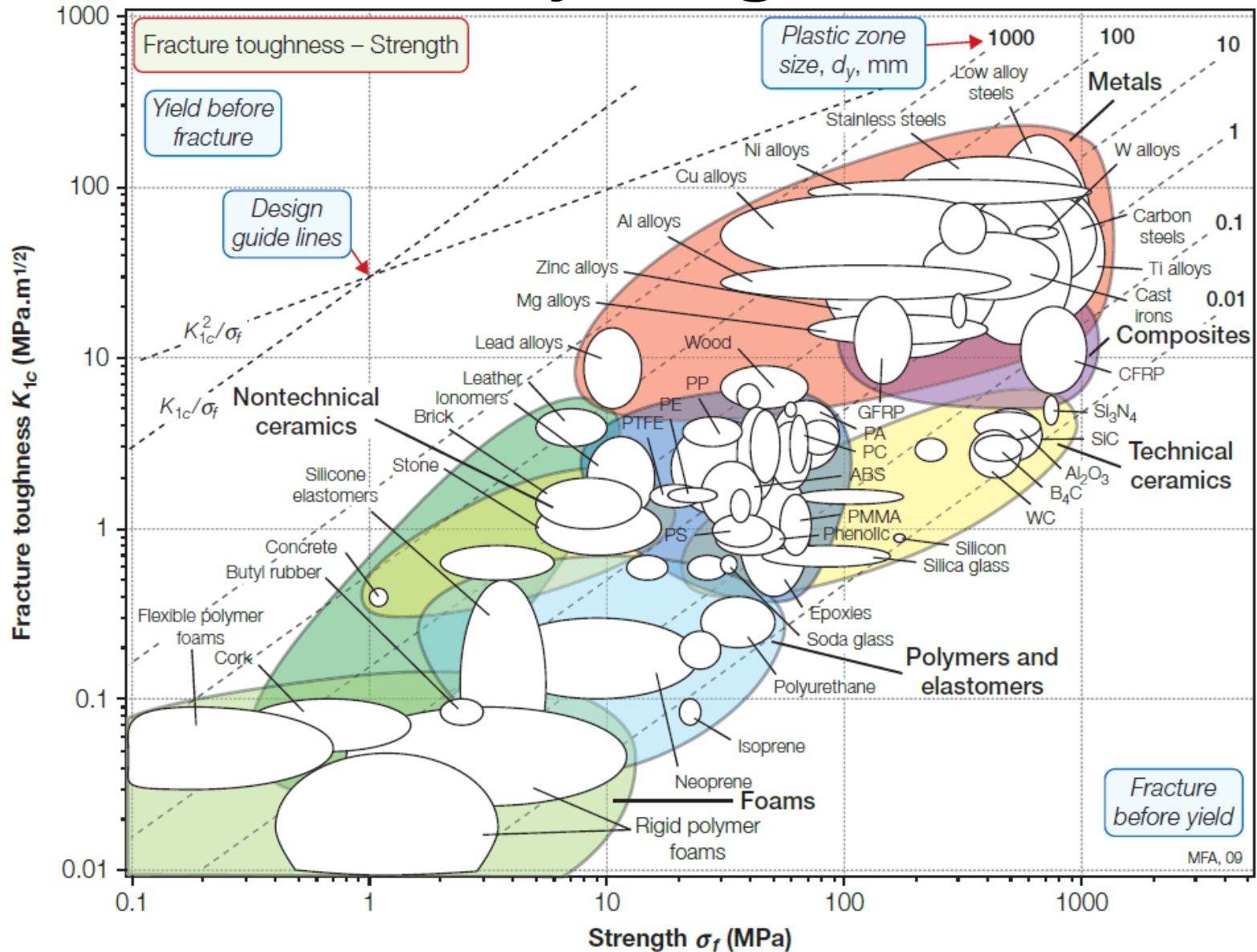


# Ashby Diagrams



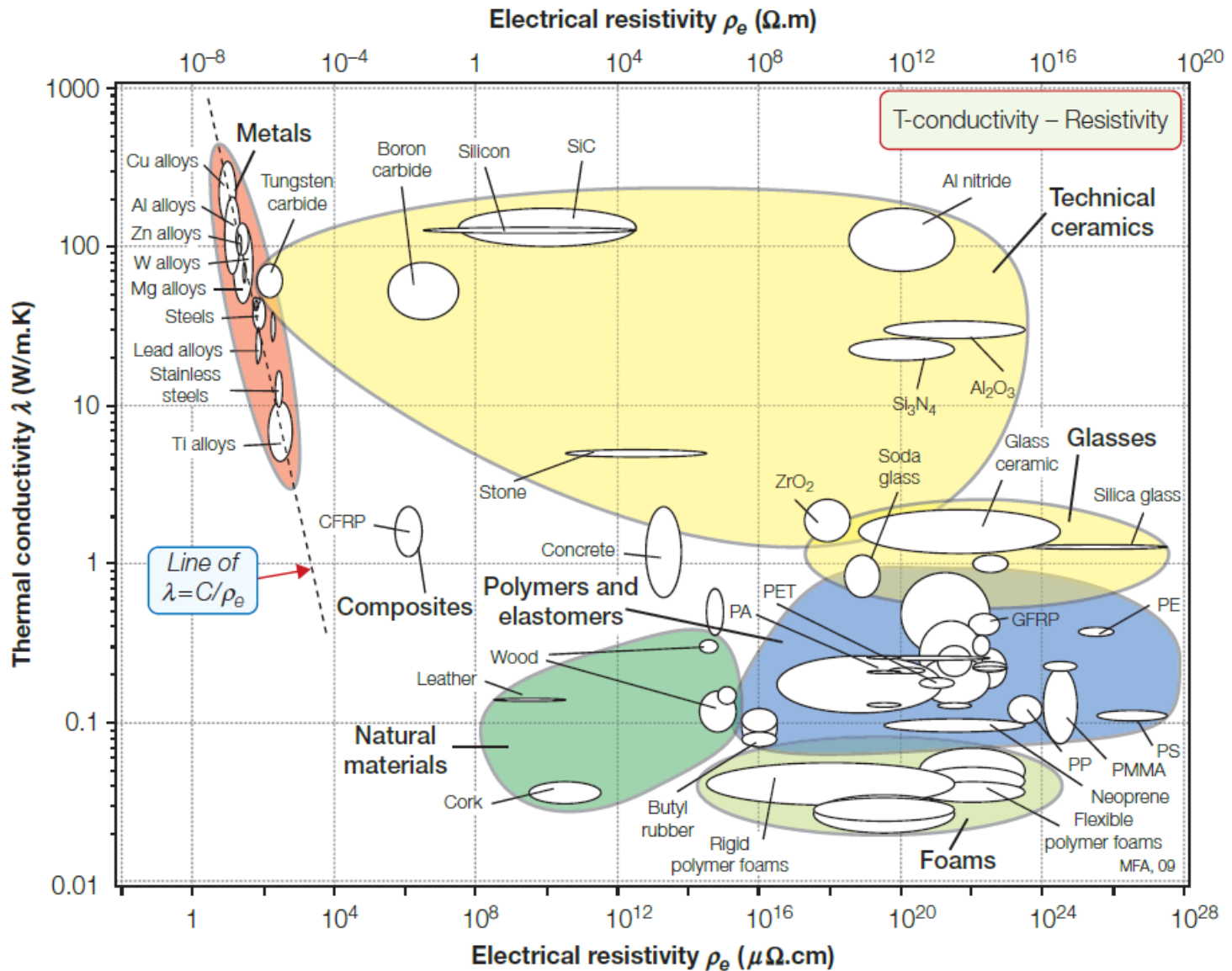


# Ashby Diagrams





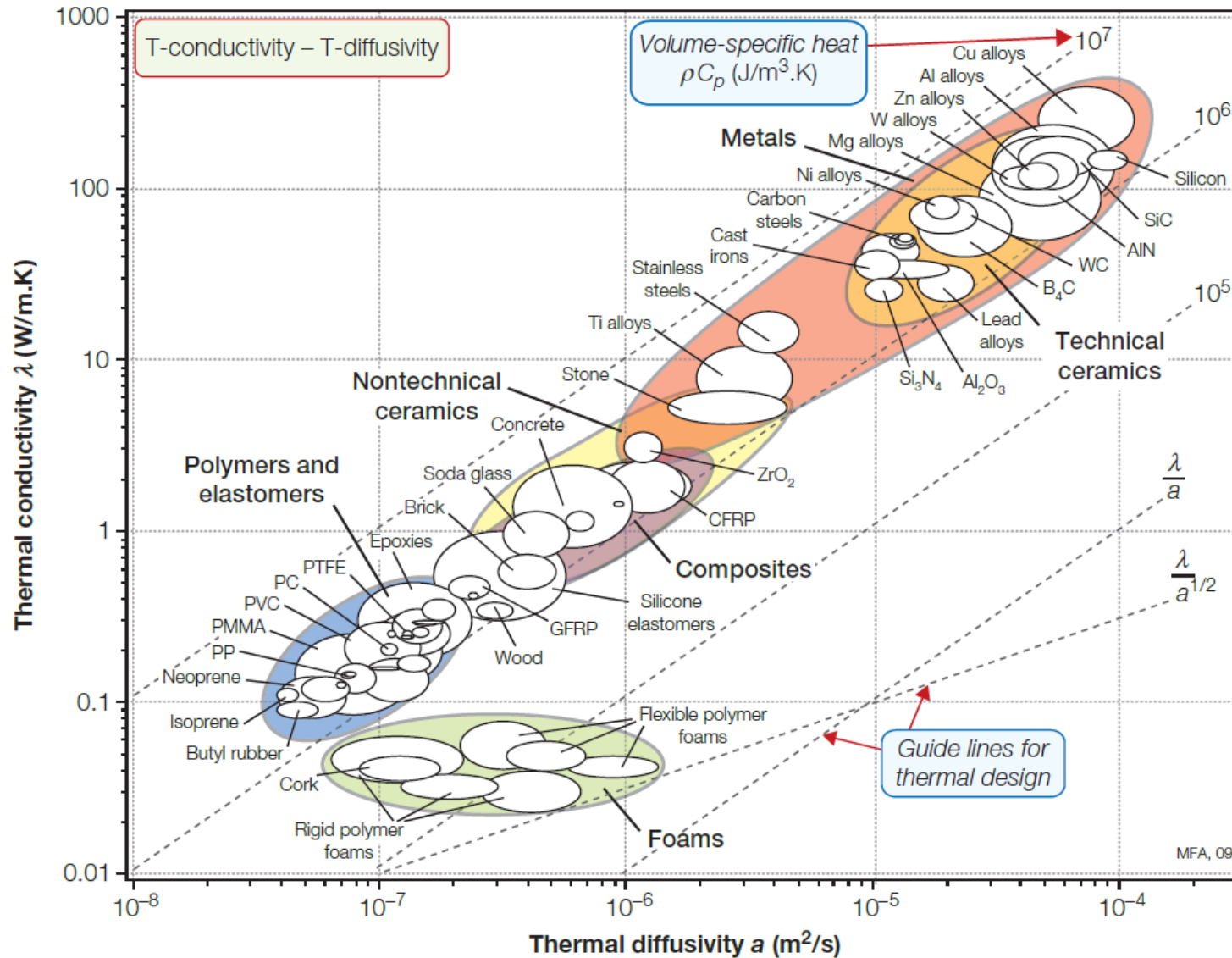
# Ashby Diagrams





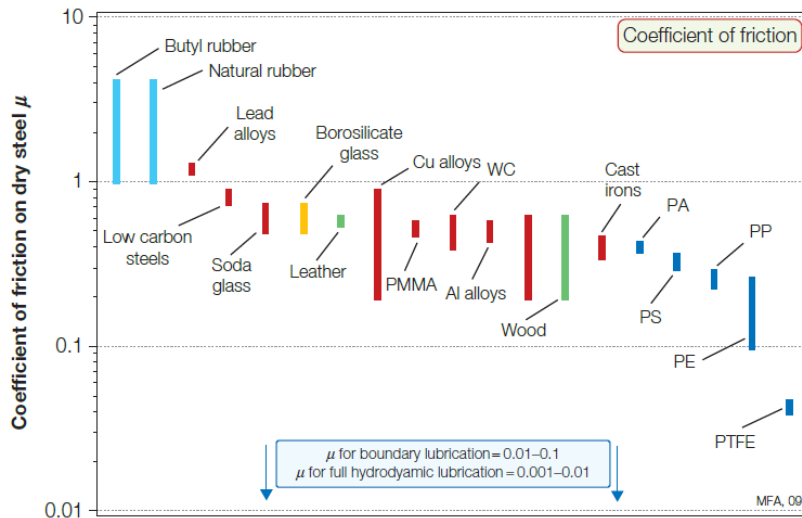
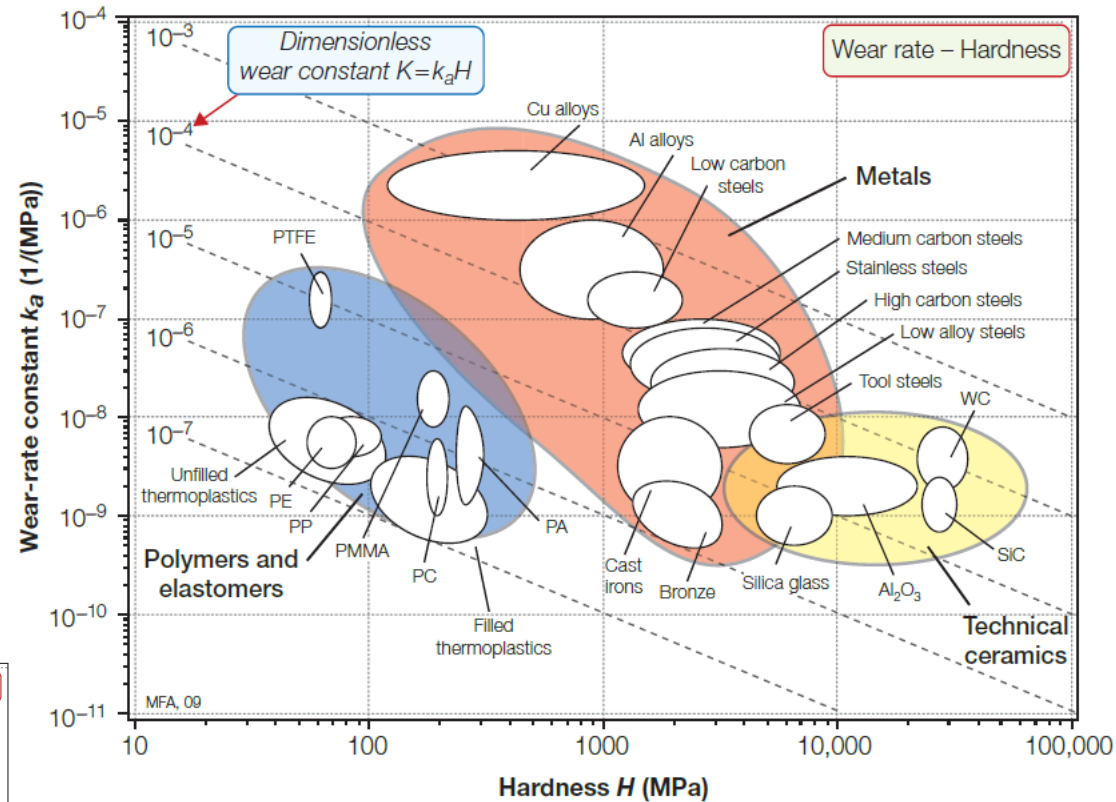


# Ashby Diagrams



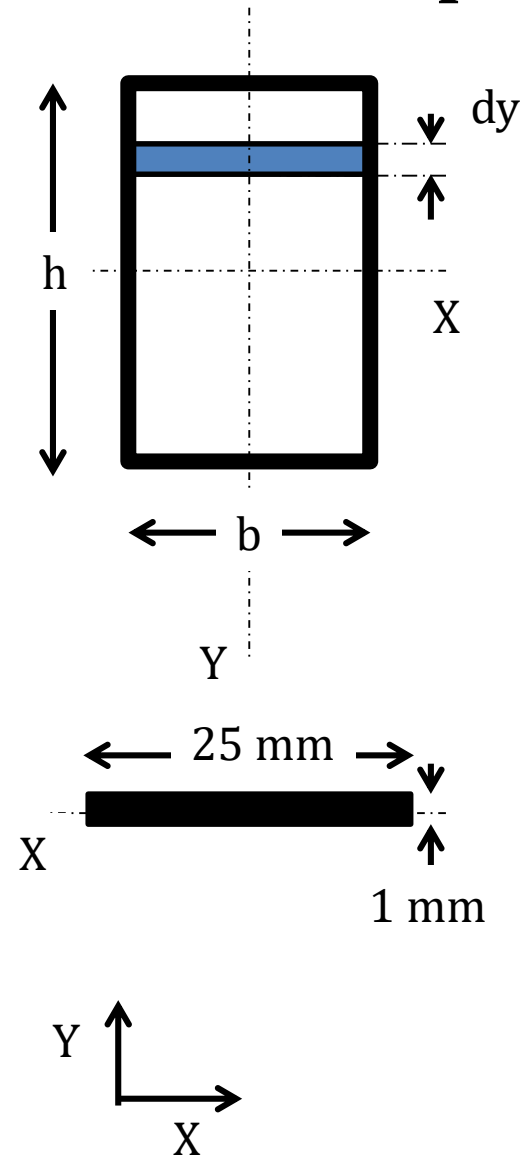


# Ashby Diagrams





# Simplification: Where is the problem?



For a beam under flexion, the moment of inertia :  $I_{XX} = \frac{bh^3}{12}$

Length (L): 300 mm  $I_{XX} = \frac{25 \cdot 1^3}{12} = 2,1 \text{ mm}^4$

Thickness (h) = 1 mm

Width (b) = 25 mm

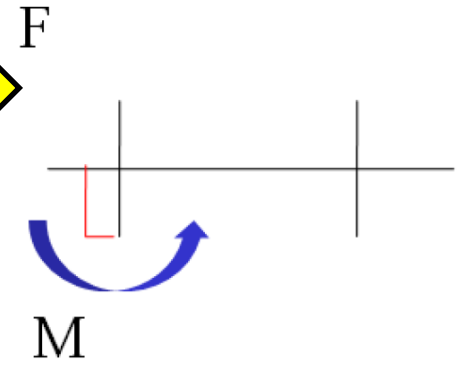
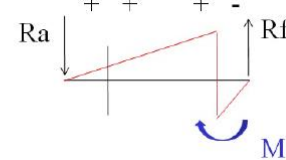
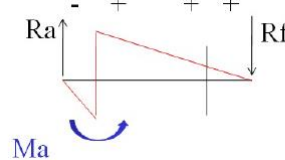
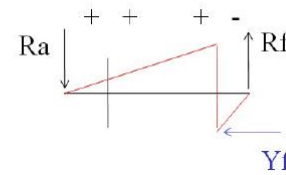
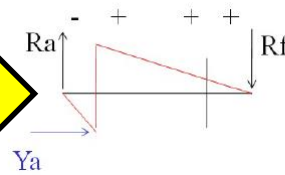
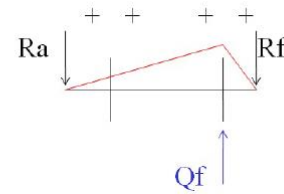
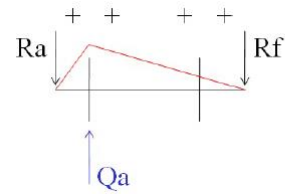
$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300 \text{ mm}^4$$

In the case of the mechanical properties, it is important to consider the forces applied, but it is the weakest point that determine the selection.

It is possible to change the geometry, but if you cannot  
What can we do?



# Simplification: Train Wheel (Fast Example)





# The Stiffness design



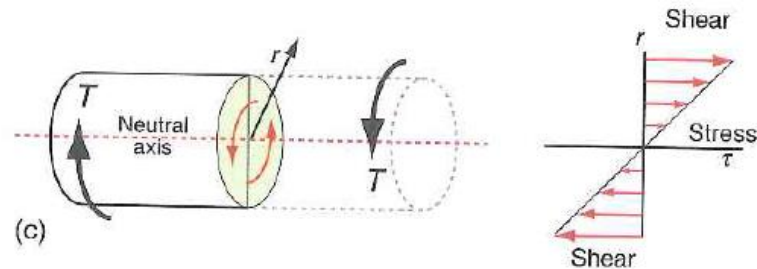
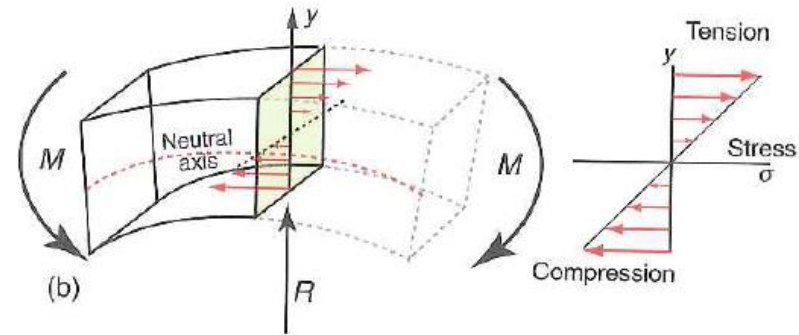
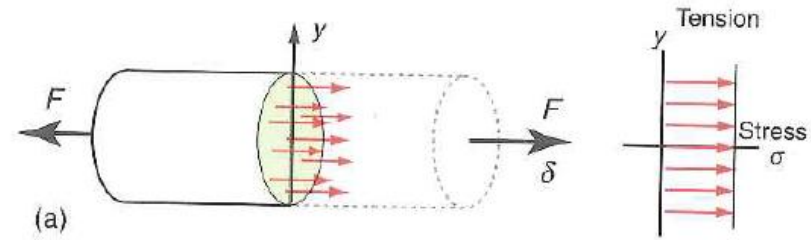
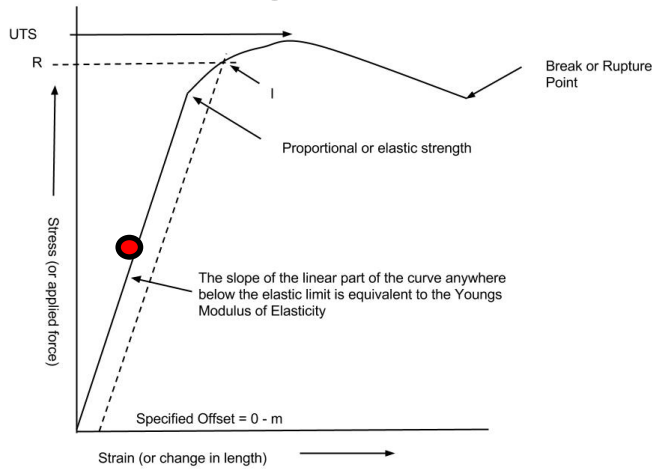
**The Stiffness design is important to avoid excessive ELASTIC deflection**



# The Stiffness design

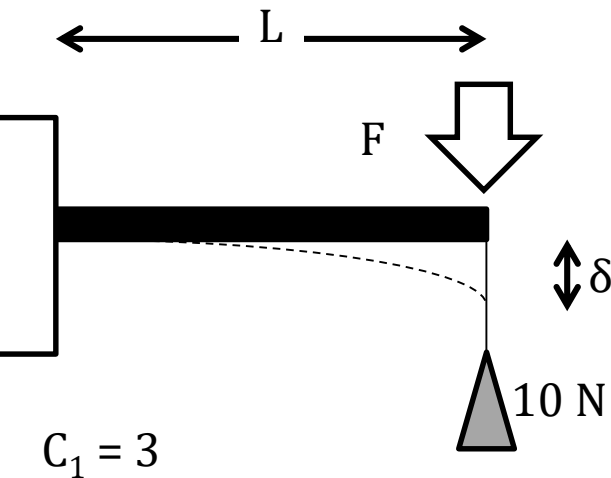
The Stiffness design is important to avoid excessive ELASTIC deflection

**● Your are here**





# The Stiffness



$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

$$\delta = \varepsilon \cdot L$$

EI = Flexural rigidity

I = Second Moment of inertia

E = Young's Modulus

$\delta$  = Deflexion

Length (L): 300 mm

Thickness (h)= 1 mm

Width (b)= 25 mm

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2,1 \text{ mm}^4$$

$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300 \text{ mm}^4$$

## Problem :

$\delta??$

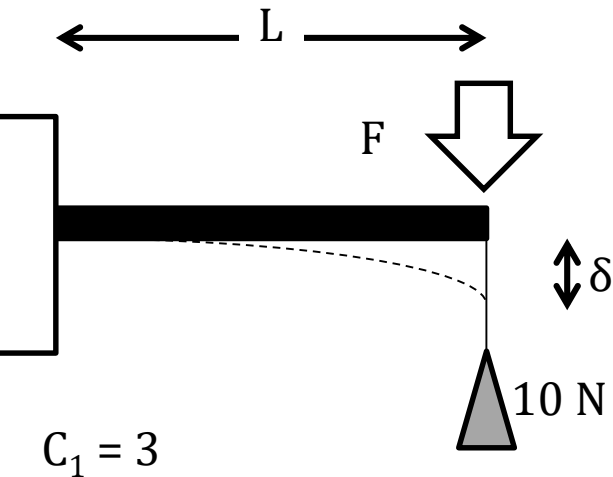
IF we consider that the beam is made of Stainless Steel (E = 200 GPa)

Which are the consequences if I want to use Polystyrene (E = 2 GPa)?

IF I can change the thickness and hold the same deflection.



# The Stiffness



$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

EI = Flexural rigidity

I = Second Moment of inertia

E = Young's Modulus

Stainless Steel ( $E = 200 \text{ GPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )  
 Polystyrene ( $E = 2 \text{ GPa}$ ;  $\rho = 1040 \text{ kg/m}^3$ )

$$I_{YY} = \frac{1 \cdot 25^3}{12} = 1300 \text{ mm}^4 \quad \rightarrow \quad \delta = \frac{10 \cdot (0,25)^3}{3 \cdot (200 \cdot 10^9) \cdot (1300 \cdot 10^{-12})} = 0,02 \text{ mm}$$

$$I_{XX} = \frac{25 \cdot 1^3}{12} = 2,1 \text{ mm}^4 \quad \rightarrow \quad \delta = \frac{FL^3}{C_1 E Y_{XX}} = 124 \text{ mm} \quad \text{☠}$$

} Steel

With  $\delta = 124 \text{ mm}$

$$I_{XX} = \frac{10 \cdot (0,25)^3}{3 \cdot (2 \cdot 10^9) \cdot (0,124)} = 210 \text{ mm}^4$$

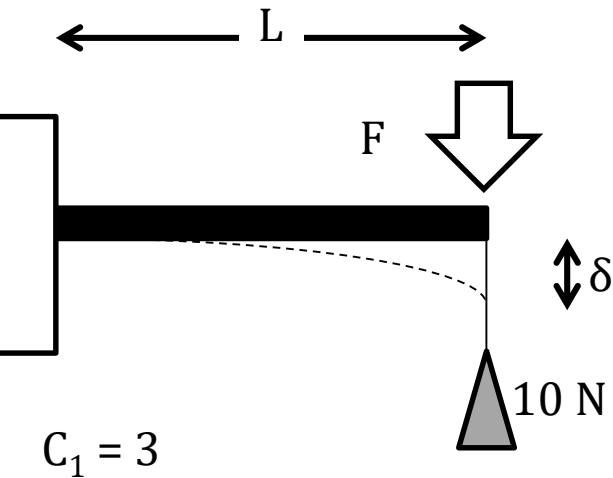
$$h = \left( \frac{12 I_{XX}}{w} \right)^{1/3} = \left( \frac{12 \cdot 210}{25} \right)^{1/3} = 4,6 \text{ mm} \quad \text{When } h(\text{Steel}) = 1 \text{ mm}$$

} PS





# The Stiffness



$$S = \frac{F}{\delta} = \frac{C_1 EI}{L^3}$$

Length: 300 mm

Width = 25 mm

EI = Flexural rigidity

I = Second Moment of inertia

E = Young's Modulus

$\delta$  = Deflexion

Stainless Steel ( $E = 200 \text{ GPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )

Polystyrene ( $E = 2 \text{ GPa}$ ;  $\rho = 1040 \text{ kg/m}^3$ )

Thickness = 1 mm

Thickness = 4,6 mm

About the weight?

$$m_{SS} = 7800 \cdot 0,3 \cdot 0,025 \cdot 0,001 = 59 \text{ gr}$$

$$m_{PS} = 1040 \cdot 0,3 \cdot 0,025 \cdot 0,046 = 36 \text{ gr}$$

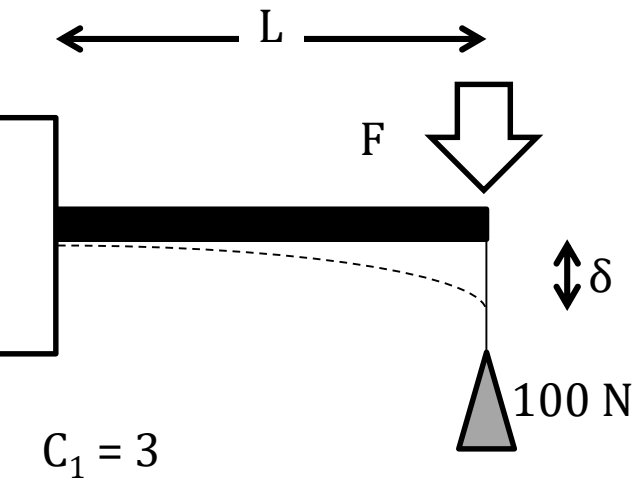
**BIGGER Section  
BUT LIGHTER**

**Depends on what you need and the conditions**



# The Materials Selection approach

**Case Study 1:**  
**Find the Lightest STIFF Beam**



Length: 300 mm

Objective	<ul style="list-style-type: none"> <li>Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Stiffness specified</li> <li>Length L</li> <li>Square shape</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Area (A) of the cross-section</li> <li>Choice of the material</li> </ul>

EI = Flexural rigidity  
 I = Second Moment of inertia  
 E = Young's Modulus  
 δ = Deflexion

Hypothesis:

$$\bullet \frac{F}{\delta} = S \geq S_{min}$$

$$\left\{ \begin{array}{l} \frac{F}{\delta} \geq S_{min} = \frac{C_1 EI}{L^3} \\ m = A \cdot L \cdot \rho \end{array} \right. \rightarrow A = \frac{m}{L \cdot \rho}$$

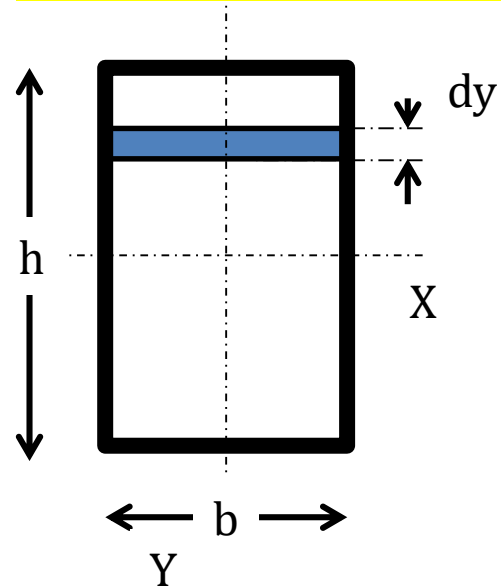
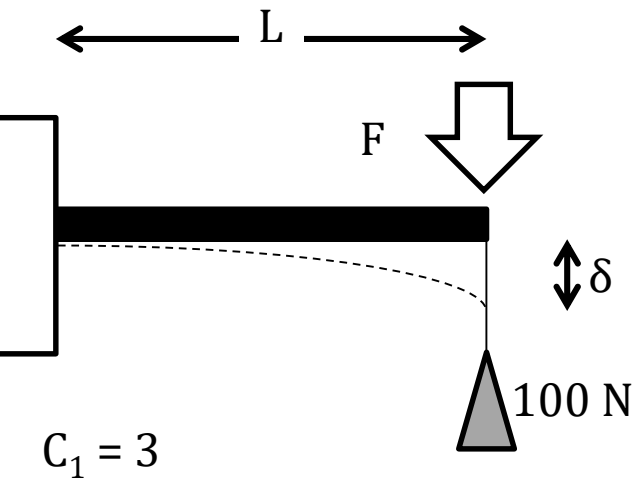
m = mass  
 A = area of the section  
 L = Length  
 ρ = Density



# The Materials Selection approach

**Case Study 1:**  
**Find the Lightest STIFF Beam**

**Beam: Square Section**  
 **$b=h$**



$$\left\{ \begin{aligned} \frac{F}{\delta} &= \frac{C_1 EI}{L^3} \geq S_{min} \\ A &= \frac{m}{L \cdot \rho} \end{aligned} \right.$$

Since  $A = b^2$

$$I = \frac{bh^3}{12} = \frac{A^2}{12}$$

$A = \frac{m}{L \cdot \rho}$     The Area will be the Free Variable  
 $m \geq \left( \frac{12 \cdot S}{C_1 \cdot L} \right)^{1/2} \cdot L^3 \cdot \frac{\rho}{E^{1/2}}$

Just remember:

Constraints	
	• Stiffness specified
	• Length L
	• Square shape



# The Material Index (M)

## Case Study 1: Find the Lightest **STIFF** Beam

$$M = \frac{A}{B}$$

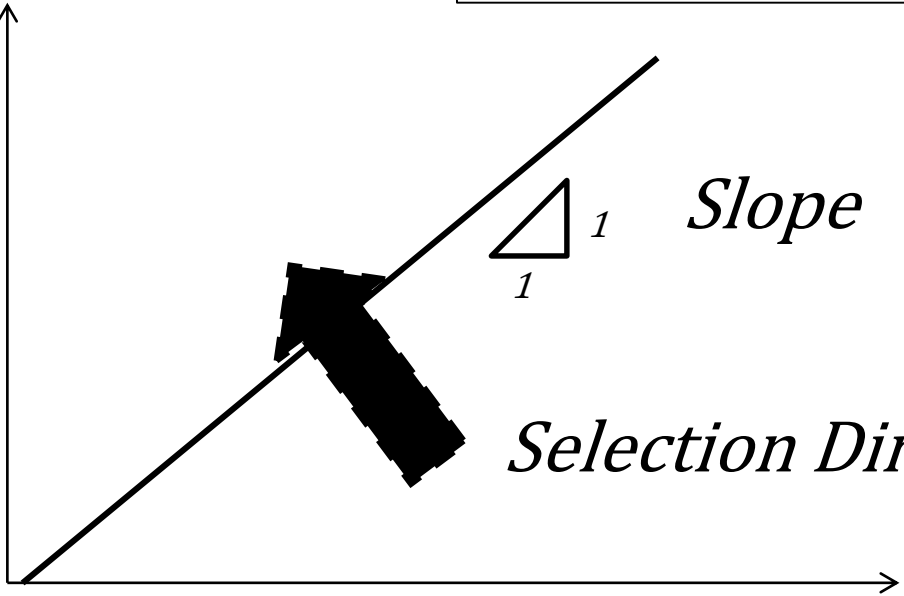


$$\text{Log}(M) = \text{Log}(A) - \text{Log}(B)$$

$$\text{Log}(A) = \text{Log}(B) + \text{Log}(M)$$

For instance  $\frac{E^{1/2}}{\rho}$

A



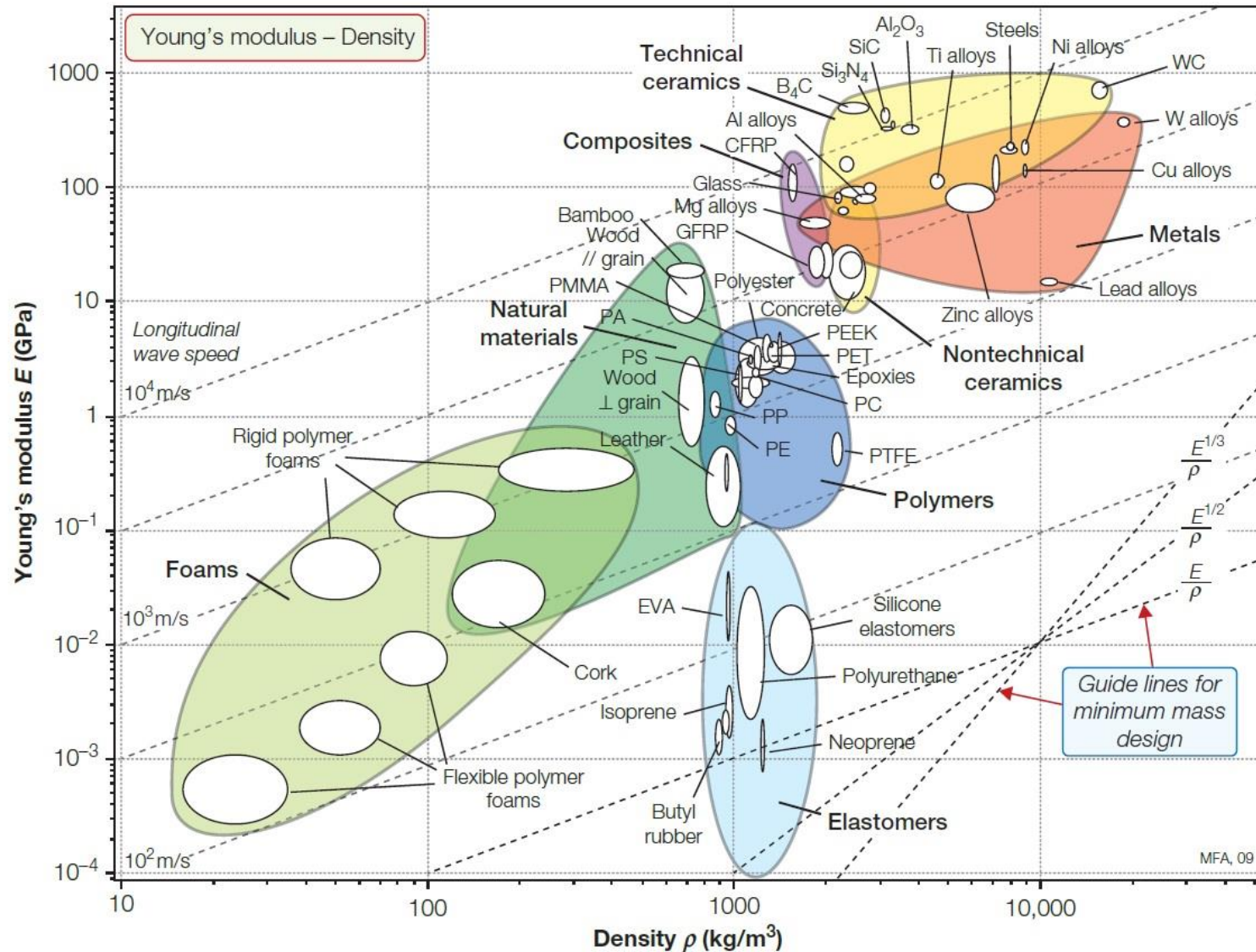
$\triangle 1$  Slope  
 $1$

*Selection Direction (+)*

B



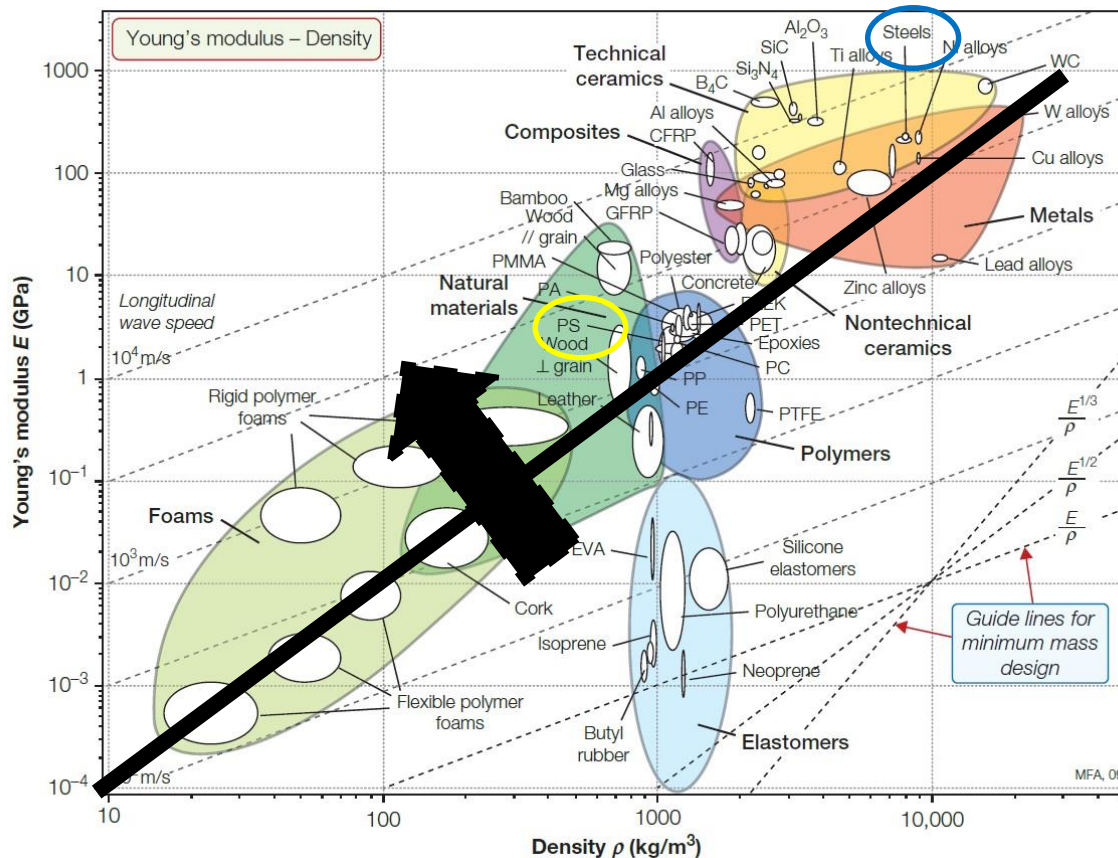
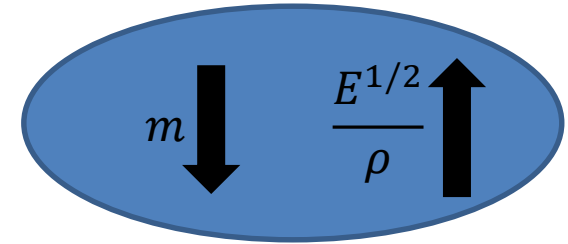
# Ashby Diagrams





# The Material Index (M)

**Case Study 1:**  
**Find the Lightest STIFF Beam**



**Stainless Steel**  
( $E = 200 \text{ GPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )  
**Polystyrene**  
( $E = 2 \text{ GPa}$ ;  $\rho = 1040 \text{ kg/m}^3$ )



# Lightest Beam (Bending conditions)

**Case Study 1:**  
**Find the Lightest STIFF Beam**

Length: 300 mm

Thickness = 1 mm

Width = 25 mm

Width and thickness ?

$$F = 100 \text{ N}$$

$$\delta = 0,34 \text{ mm}$$

$$S_{\min} = 296 \cdot 10^3 \text{ N/m}$$

Stainless Steel ( $E = 200 \text{ GPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )  
Polystyrene ( $E = 2 \text{ GPa}$ ;  $\rho = 1040 \text{ kg/m}^3$ )

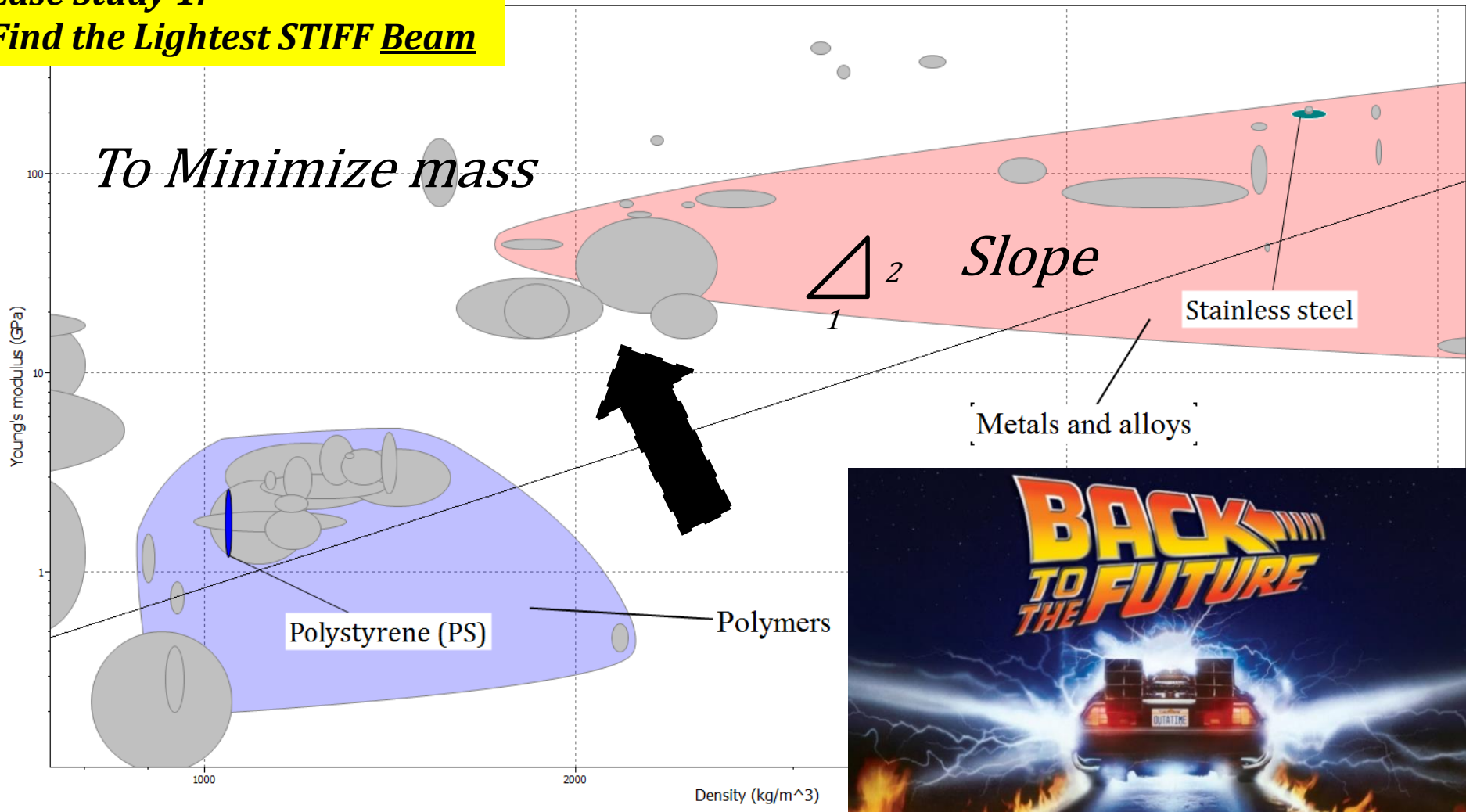
$$\left\{ \begin{array}{l} m \geq \left( \frac{12 \cdot S}{C_1 \cdot L} \right)^{1/2} \cdot L^3 \cdot \frac{\rho}{E^{1/2}} \\ A = \frac{m}{L \cdot \rho} \end{array} \right.$$

Material	Weight (kg)	A (mm <sup>2</sup> )	Width and Thickness (mm)
Stainless Steel	0,935	400	20
Polystyrene	1,25	4000	63



# CES

## Case Study 1: Find the Lightest **STIFF** Beam

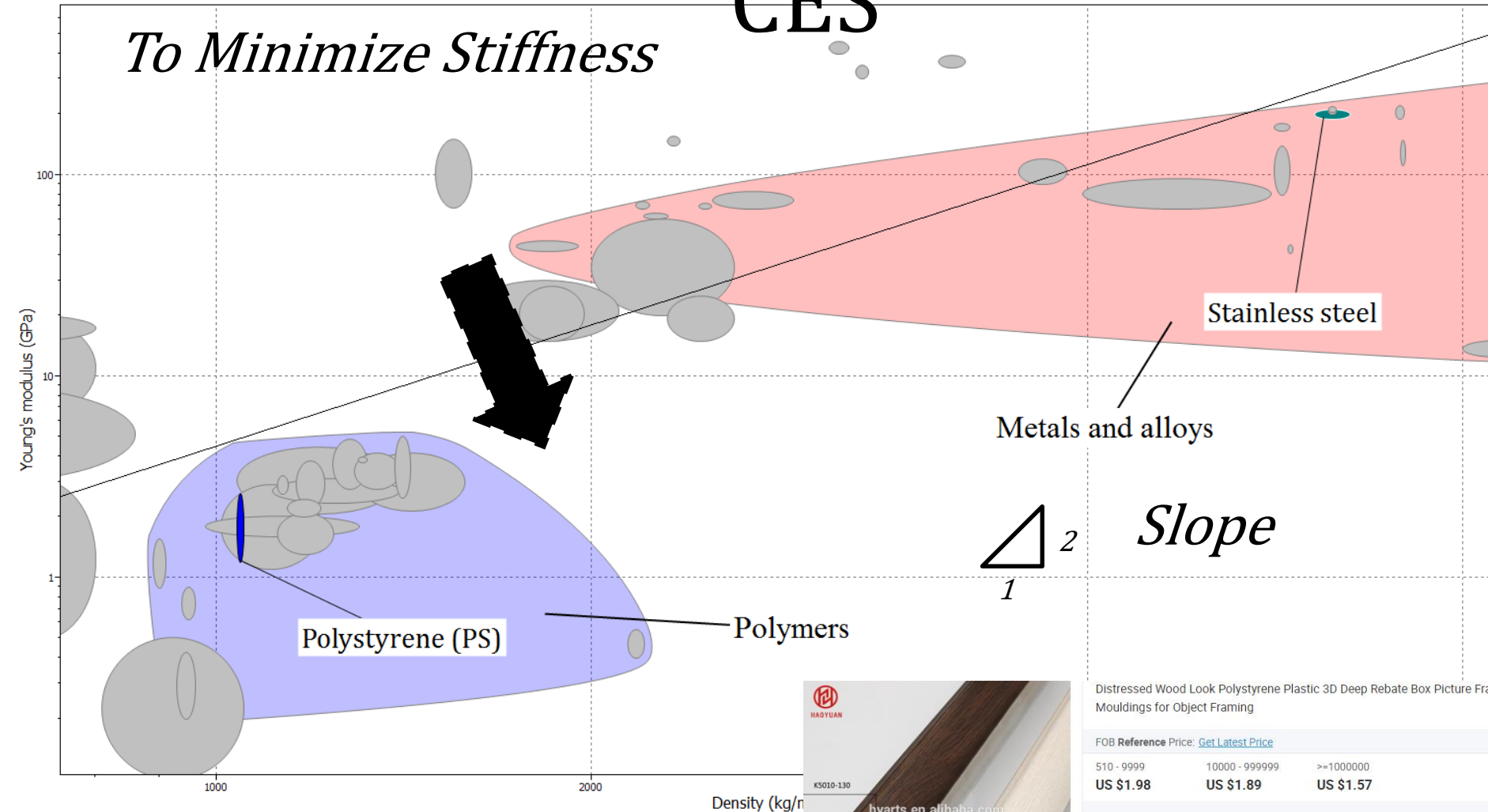






# CES

*To Minimize Stiffness*



*Depends on what you need →*



Distressed Wood Look Polystyrene Plastic 3D Deep Rebate Box Picture Frame Mouldings for Object Framing

FOB Reference Price: [Get Latest Price](#)

510 - 9999	10000 - 999999	>=1000000
US \$1.98	US \$1.89	US \$1.57

Supply Ability: 2000 Meter/Meters per Day 3D picture frame moulding

Port: Ningbo

[Contact Supplier](#)

[Start Order](#)

[Chat Now!](#)

*It is not in flexion*



# Ok, slow down..

**Case Study 2:**  
**Find the Lightest STIFF Tie-Rod**

Objective	<ul style="list-style-type: none"> <li>Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Stiffness specified</li> <li>Length L</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Area (A) of the cross-section</li> <li>Choice of the material</li> </ul>

Tie-Rod =

## TRACTION CONDITIONS

### DATA

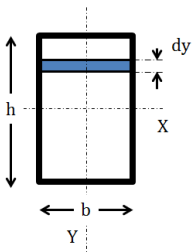
F = 1000 N

### Dimensions:

Length: 300 mm

Thickness = 1 mm

Width = 25 mm



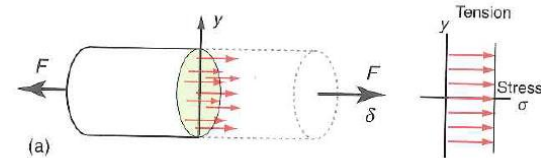
**In Traction,**  
**the shape of the cross-section is not important**

$$m = A \cdot L \cdot \rho \longrightarrow A = \frac{m}{L \cdot \rho}$$

From material:  $\frac{\sigma}{\epsilon} = E$

From definition:  $\delta = \epsilon \cdot L$

$$F = \sigma \cdot A$$



$$\frac{F}{\delta} \geq S_{min} = S$$



# Lightest Tie-Rod (Traction conditions)

**Case Study 2:**  
**Find the Lightest STIFF Tie-Rod**

$$\frac{F}{\delta} \geq S_{min} = S$$

$$F = 1000 \text{ N}$$
$$\delta = 3,78 \cdot 10^{-3} \text{ mm}$$
$$S_{min} = 264,5 \cdot 10^6 \text{ N/m}$$

**Dimensions:**

Length: 300 mm

Thickness = 1 mm

Width = 25 mm

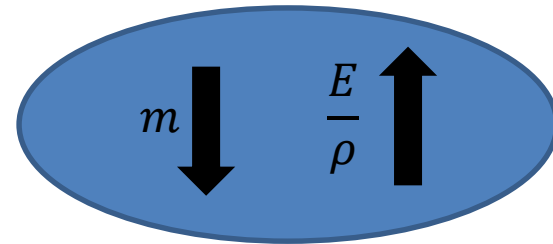
$$\frac{\sigma \cdot A}{\varepsilon \cdot L} \geq S_{min}$$

$$\frac{E \cdot A}{L} \geq S_{min}$$

$$A = \frac{m}{L \cdot \rho}$$

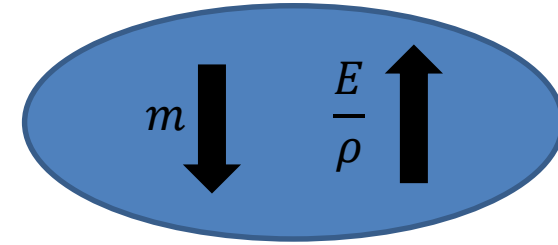
$$m \geq S \cdot L^2 \cdot \frac{\rho}{E}$$

$$m \geq (264,5 \cdot 10^6) \cdot (300 \cdot 10^{-3})^2 \cdot \frac{\rho}{E}$$
$$\geq \dots \cdot \frac{\rho}{E}$$

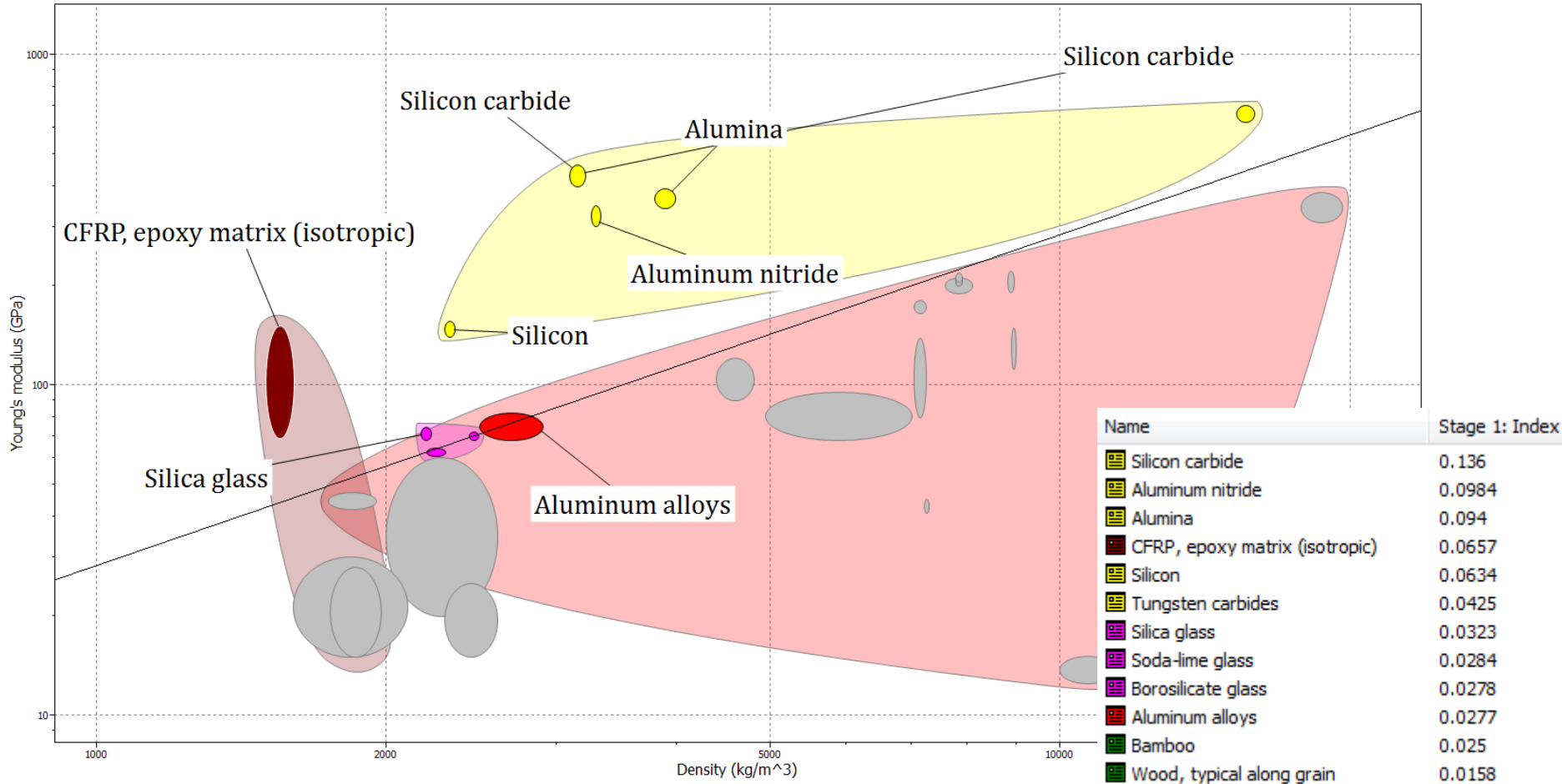




# CES



**Case Study 2:**  
**Find the Lightest STIFF Tie-Rod**





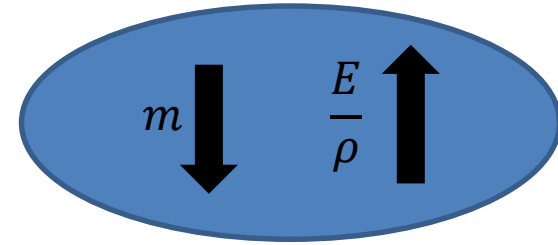
# Lightest Tie-Rod (Traction conditions)

**Case Study 2:**  
**Find the Lightest STIFF Tie-Rod**

$$\frac{E \cdot A}{L} \geq S_{min}$$

$$A = \frac{m}{L \cdot \rho}$$

$$m \geq S \cdot L^2 \cdot \frac{\rho}{E}$$



$$F = 1000 \text{ N}$$

$$\delta = 3,78 \cdot 10^{-3} \text{ mm}$$

$$S_{min} = 264,5 \cdot 10^6 \text{ N/m}$$

## Dimensions:

Length: 300 mm

Thickness = 1 mm

Width = 25 mm

Stainless Steel (E = 200 GPa;  $\rho = 7800 \text{ kg/m}^3$ )  
Silicon carbide (E = 430 GPa;  $\rho = 3150 \text{ kg/m}^3$ )  
Al Alloys (E = 75 GPa;  $\rho = 2700 \text{ kg/m}^3$ )

Material	Weight (kg)	A (mm <sup>2</sup> )	Width and Thickness (mm)
Silicon Carbide	0,174	179,8	13,4
Al Alloys	0,856	1050	32,4



# Change of the section

**Panel:**

b fixed

h free

**Beam: Square**  
**Section**

$b=h$



**Panel:**

h fixed

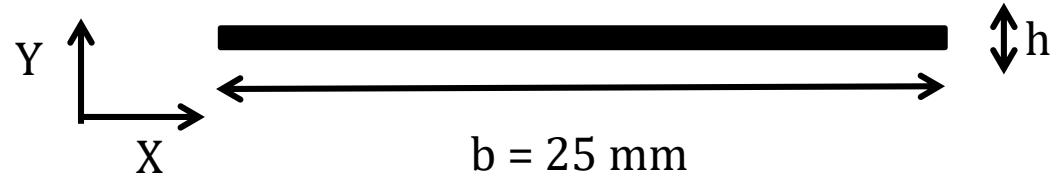
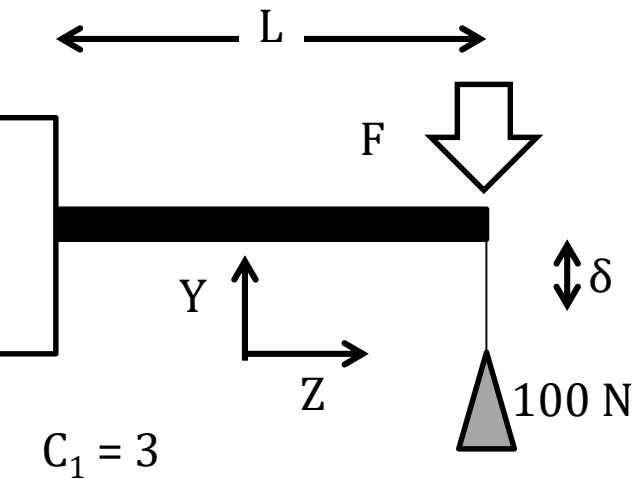
b free



# Lightest Panel (Bending conditions)

**Case Study 3:**  
**Find the Lightest STIFF Panel**

Objective	<ul style="list-style-type: none"> <li>Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Stiffness specified</li> <li>Length L and b specified</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>h (thickness) of the cross-section</li> <li>Choice of the material</li> </ul>



Hypothesis:

- $\frac{F}{\delta} \geq S = S_{min}$

Length: 300 mm

Width: 25 mm

$$\left\{ \begin{array}{l} \frac{F}{\delta} \geq \frac{C_1 EI}{L^3} = S_{min} \\ m = A \cdot L \cdot \rho \end{array} \right. \rightarrow A = \frac{m}{L \cdot \rho}$$

m = mass  
 A = area of the section  
 L = Length  
 ρ = Density

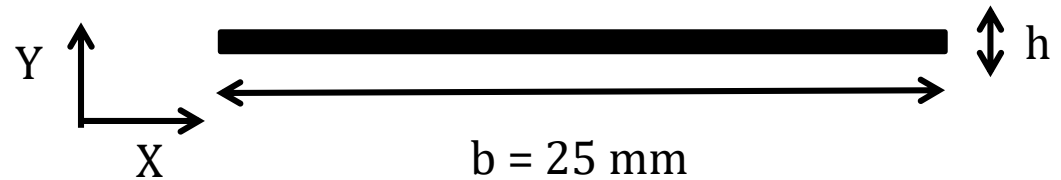


# Lightest Panel (Bending conditions)

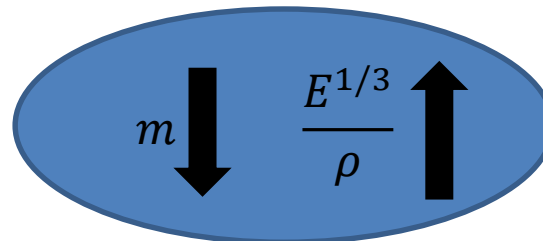
**Case Study 3:**  
**Find the Lightest STIFF Panel**

**Panel:**  
**b fixed**  
**h free**

Since  $A = bh$   $\Rightarrow$   $h = \frac{A}{b}$   $\Rightarrow$   $I = \frac{b \cdot \left(\frac{A}{b}\right)^3}{12} = \frac{A^3}{12 \cdot b^2}$   $\Rightarrow$   $\left\{ \begin{array}{l} S_{min} \leq \frac{C_1 EI}{L^3} \\ A = \frac{m}{L \cdot \rho} \end{array} \right.$



$\left\{ \begin{array}{l} A = \frac{m}{L \cdot \rho} \\ m \geq \left( \frac{12 \cdot S \cdot b^2}{C_1} \right)^{1/3} \cdot L^2 \cdot \frac{\rho}{E^{1/3}} \end{array} \right.$  The Area will be the Free Variable, but all the consequences of the selection are on the thickness  $h = \frac{A}{b}$

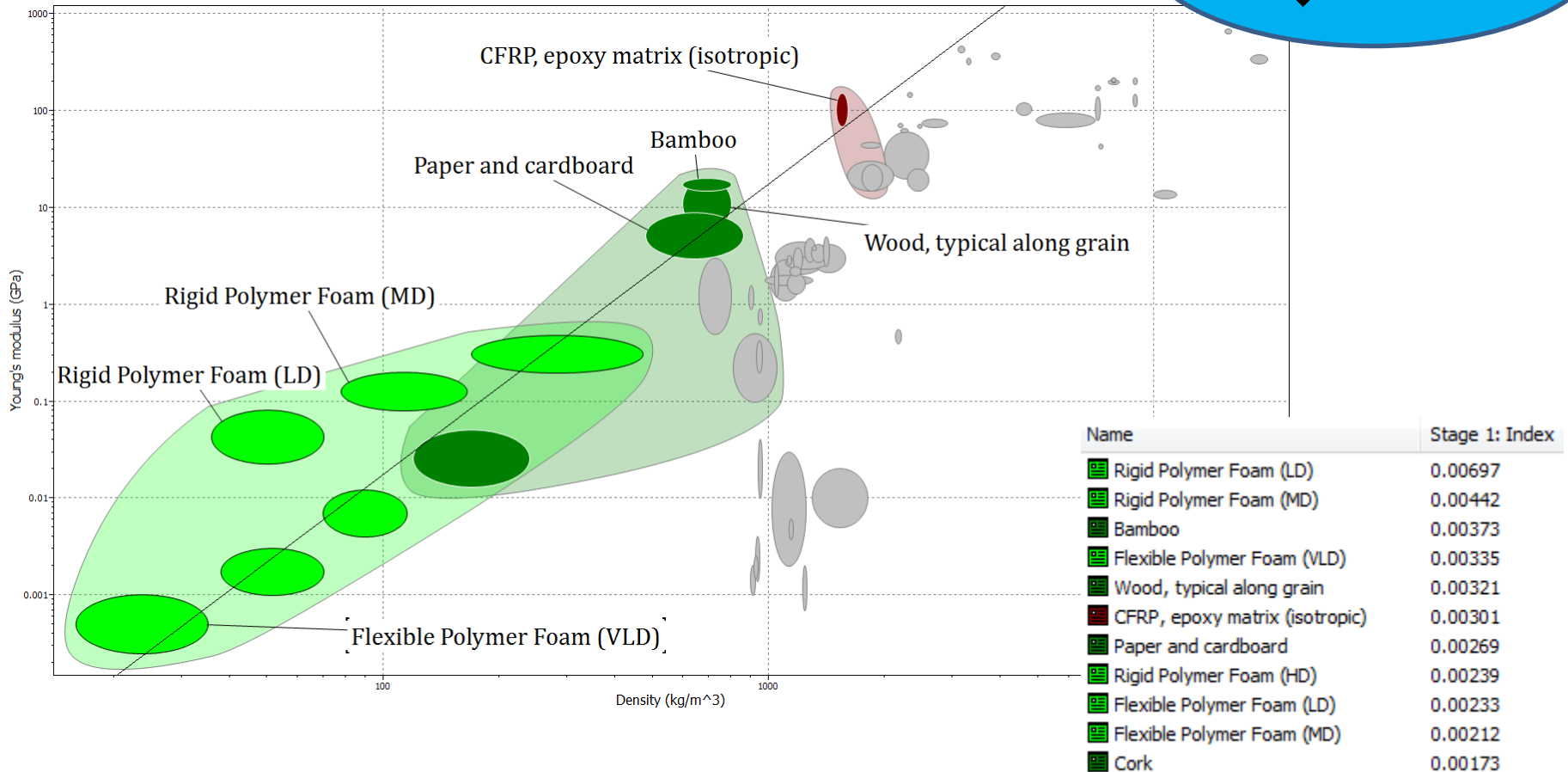
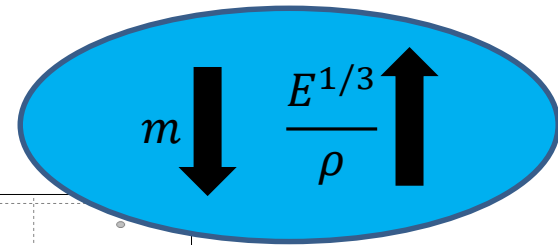






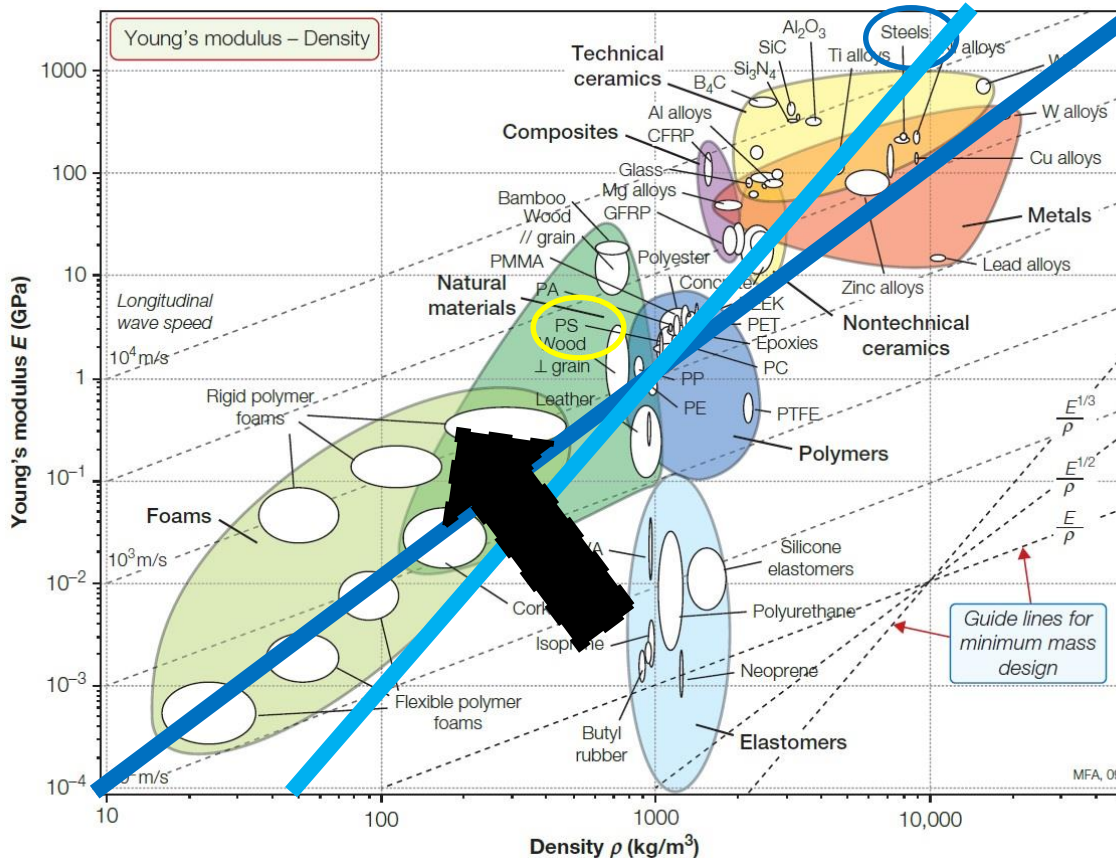
# CES

## Case Study 3: Find the Lightest STIFF Panel





# Bending conditions



$$m \downarrow \quad \frac{E^{1/2}}{\rho} \uparrow$$

$$m \downarrow \quad \frac{E^{1/3}}{\rho} \uparrow$$

**Stainless Steel**  
( $E = 200 \text{ GPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )  
**Polystyrene**  
( $E = 2 \text{ GPa}$ ;  $\rho = 1040 \text{ kg/m}^3$ )



# Bending conditions

$$F = 100 \text{ N}$$

$$\delta = 0,34 \text{ mm}$$

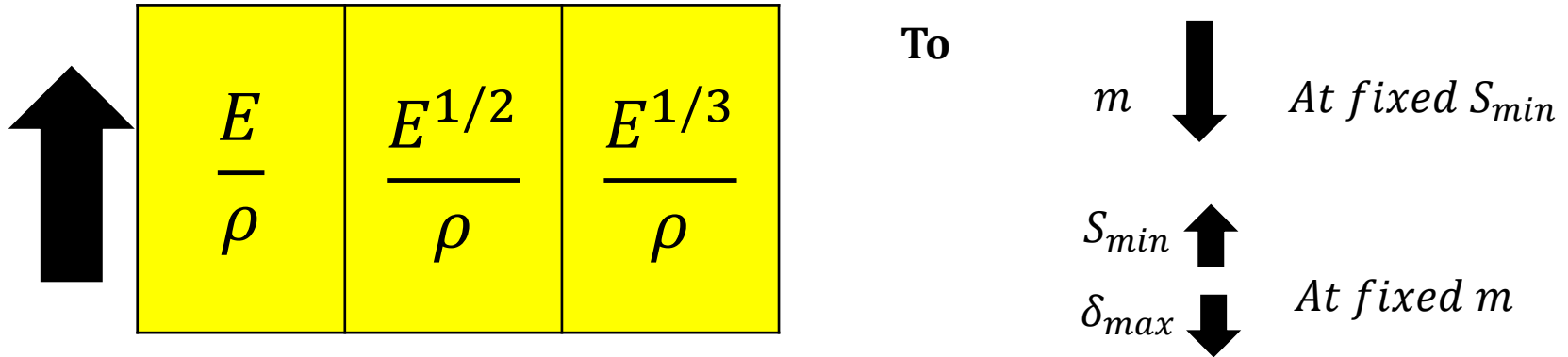
$$S_{\min} = 296 \cdot 10^3 \text{ N/m}$$

Stainless Steel ( $E = 200 \text{ GPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )  
Polystyrene ( $E = 2 \text{ GPa}$ ;  $\rho = 1040 \text{ kg/m}^3$ )

	Material	Weight (kg)	A (mm <sup>2</sup> )	Thickness h (mm)
Beam	Stainless Steel	0,935	400	20
	Polystyrene	1,25	4000	63
Panel (b= 25 mm)	Stainless Steel	1,09	466	21,59
	Polystyrene	0,67	2147	46,34



# Stiffness Summary



Stiffness – Traction :

Name	Stage 1: Index
Silicon carbide	0.136
Aluminum nitride	0.0984
Alumina	0.094
CFRP, epoxy matrix (isotropic)	0.0657
Silicon	0.0634
Tungsten carbides	0.0425
Silica glass	0.0323
Soda-lime glass	0.0284
Borosilicate glass	0.0278
Aluminum alloys	0.0277
Bamboo	0.025
Wood, typical along grain	0.0158

Stiffness – Bending :

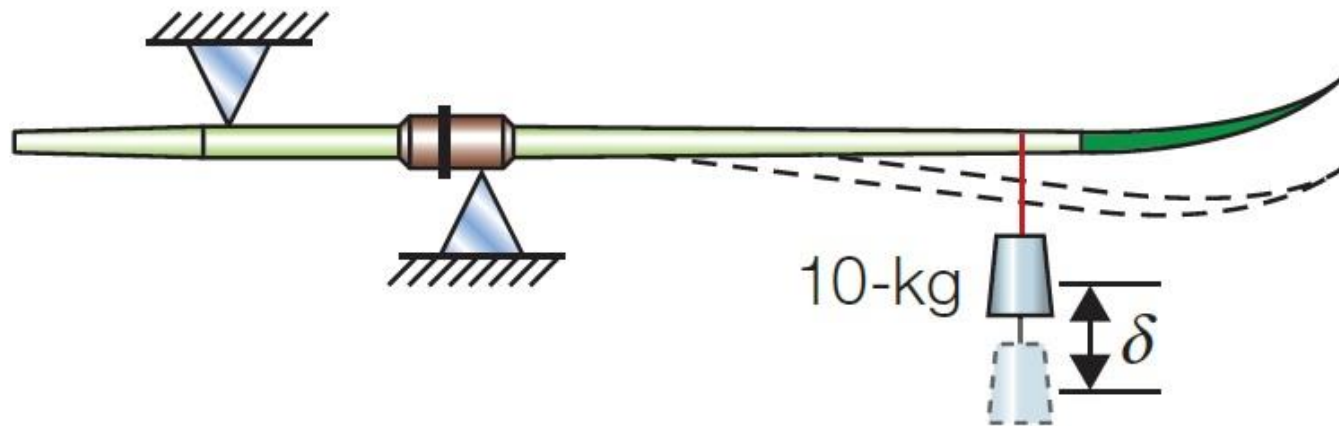
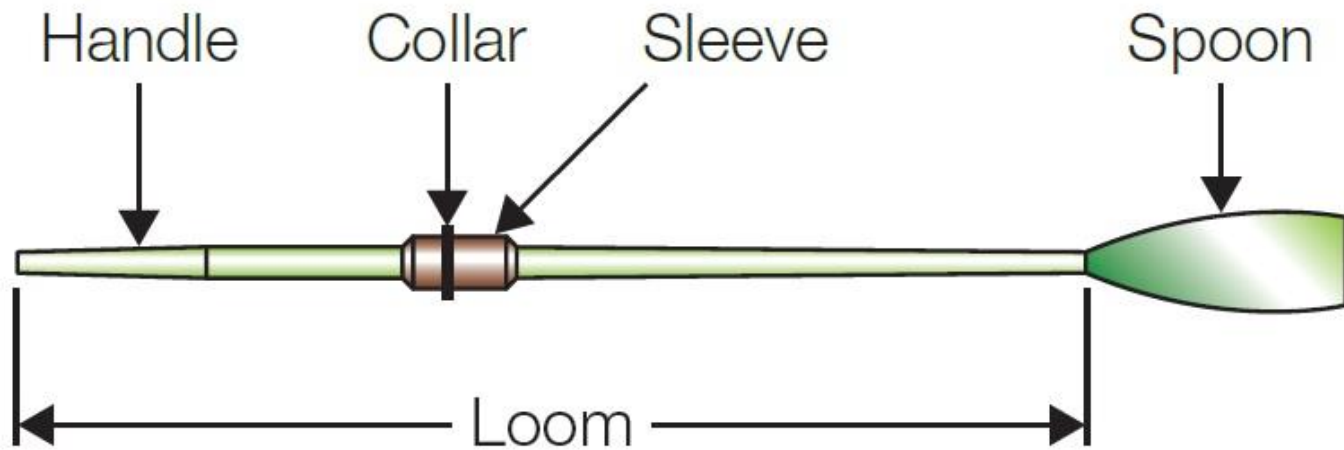
Name	Stage 1: Index
Silicon carbide	0.00657
CFRP, epoxy matrix (isotropic)	0.00651
Bamboo	0.00601
Aluminum nitride	0.00546
Silicon	0.00522
Alumina	0.00492
Wood, typical along grain	0.00478
Rigid Polymer Foam (LD)	0.00413
Silica glass	0.00384
Magnesium alloys	0.00362
Paper and cardboard	0.00354
Rigid Polymer Foam (MD)	0.00314

Stiffness – Bending :

Name	Stage 1: Index
Rigid Polymer Foam (LD)	0.00697
Rigid Polymer Foam (MD)	0.00442
Bamboo	0.00373
Flexible Polymer Foam (VLD)	0.00335
Wood, typical along grain	0.00321
CFRP, epoxy matrix (isotropic)	0.00301
Paper and cardboard	0.00269
Rigid Polymer Foam (HD)	0.00239
Flexible Polymer Foam (LD)	0.00233
Flexible Polymer Foam (MD)	0.00212
Cork	0.00173

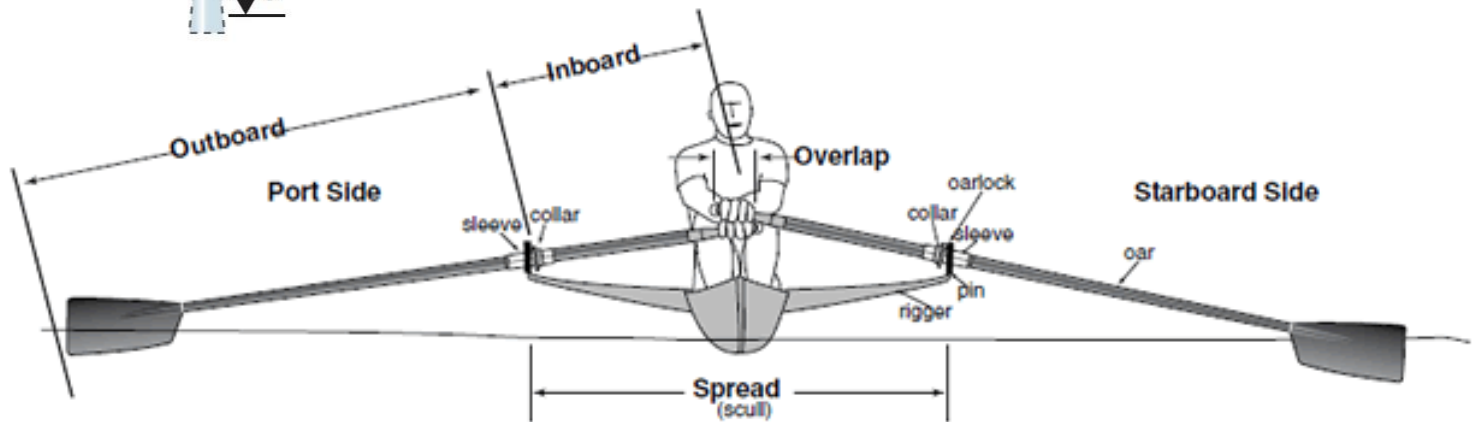
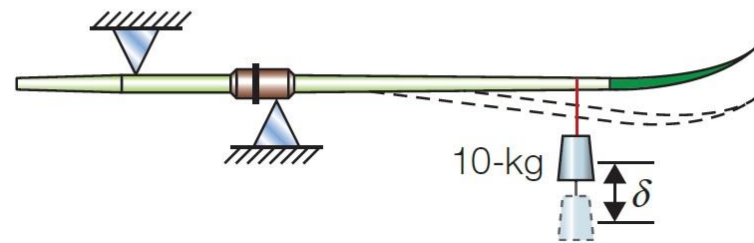
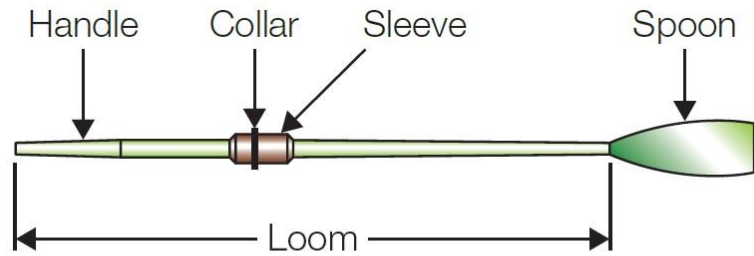


**Case Study 4:  
Materials for Oars**





## Case Study 4: Materials for Oars

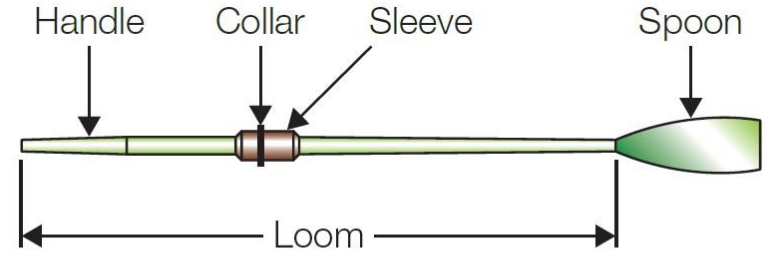


$L$  (Outboard) = 2 m

Objective	<ul style="list-style-type: none"> <li>Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Stiffness specified</li> <li>Length <math>L</math></li> <li>Circular shape (beam)</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Area (<math>A</math>) of the cross-section</li> <li>Choice of the material</li> </ul>

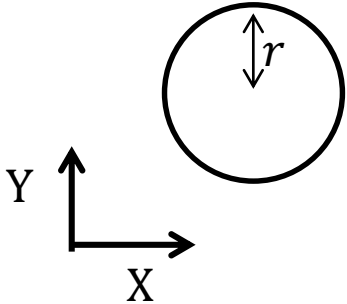


# Case Study 4: Materials for Light Oars

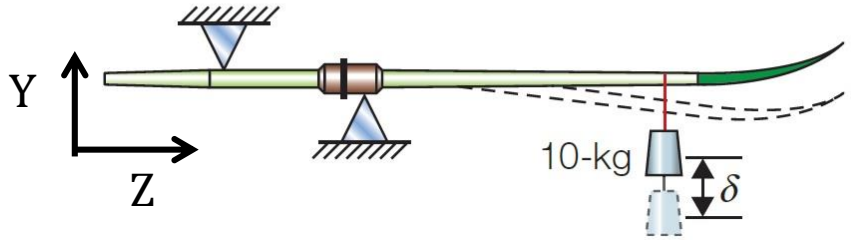


We assume solid section

$$A = \pi \cdot r^2$$

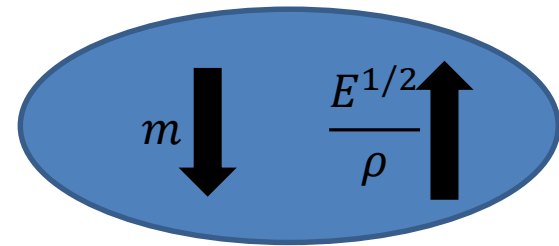


$$I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi}$$



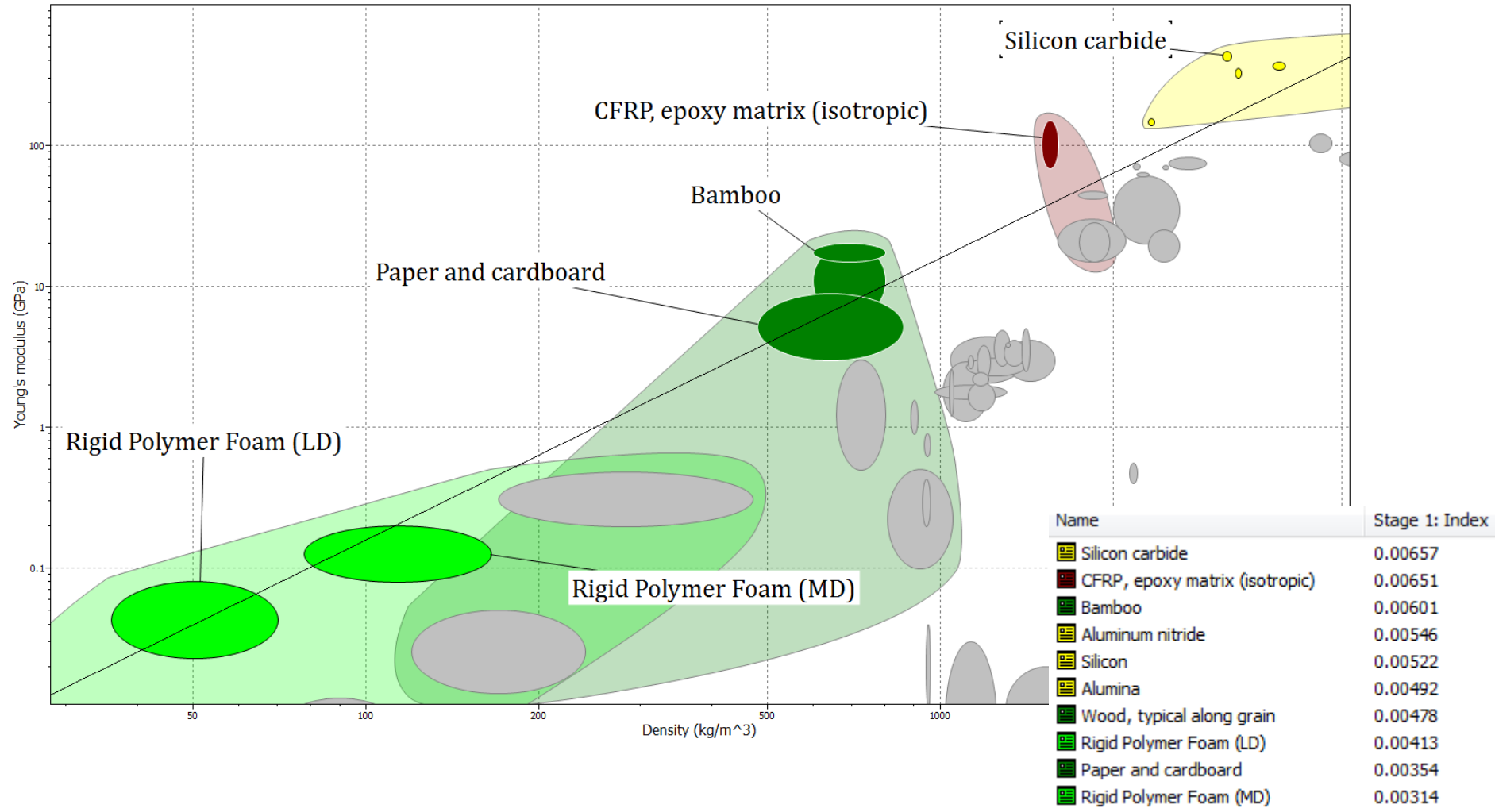
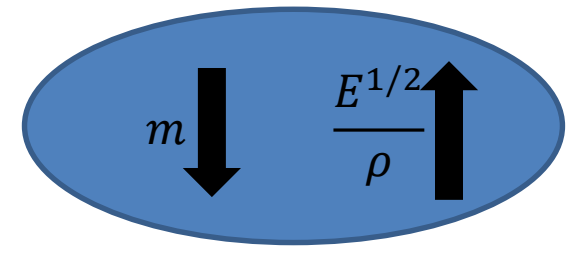
$$\left\{ \begin{array}{l} \frac{F}{\delta} \geq S_{min} = \frac{C_1 EI}{L^3} \\ A = \frac{m}{L \cdot \rho} \end{array} \right.$$

$$\left\{ \begin{array}{l} A = \frac{m}{L \cdot \rho} \\ m \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho}{E^{1/2}} \end{array} \right.$$





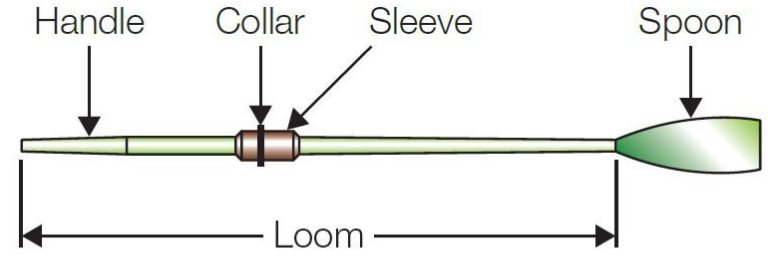
# Case Study 4: Materials for Light Oars







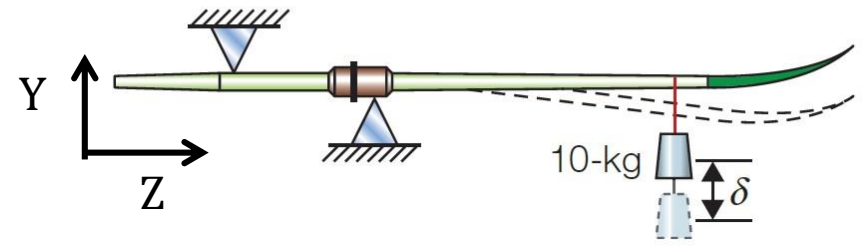
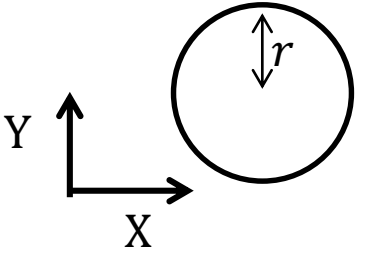
# Case Study 4: Materials for Light and Slender Oars



We assume solid section

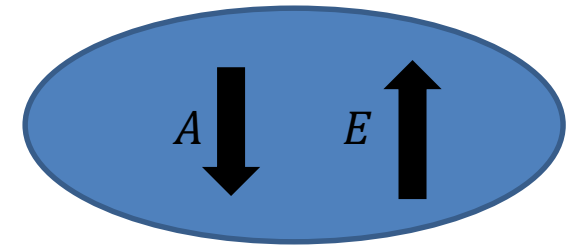
$$A = \pi \cdot r^2$$

$$I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi}$$



$$\left\{ \begin{array}{l} \frac{F}{\delta} \geq S_{min} = \frac{C_1 EI}{L^3} \\ I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi} \end{array} \right.$$

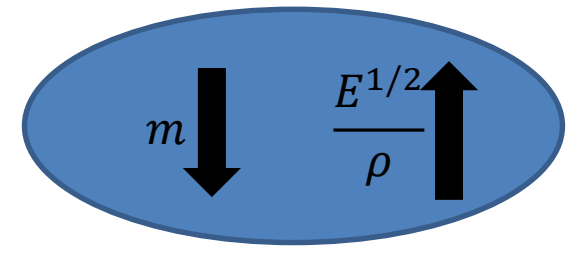
$$A \leq \left( \frac{4 \cdot \pi \cdot S \cdot L^3}{3} \right)^{1/2} \cdot \frac{1}{E^{1/2}}$$



Place LIMITS to a single Property  
Evaluating the Properties Chart  $\longrightarrow$  10 Gpa < E < 200 GPa



# Case Study 4: Materials for Light and Slender Oars

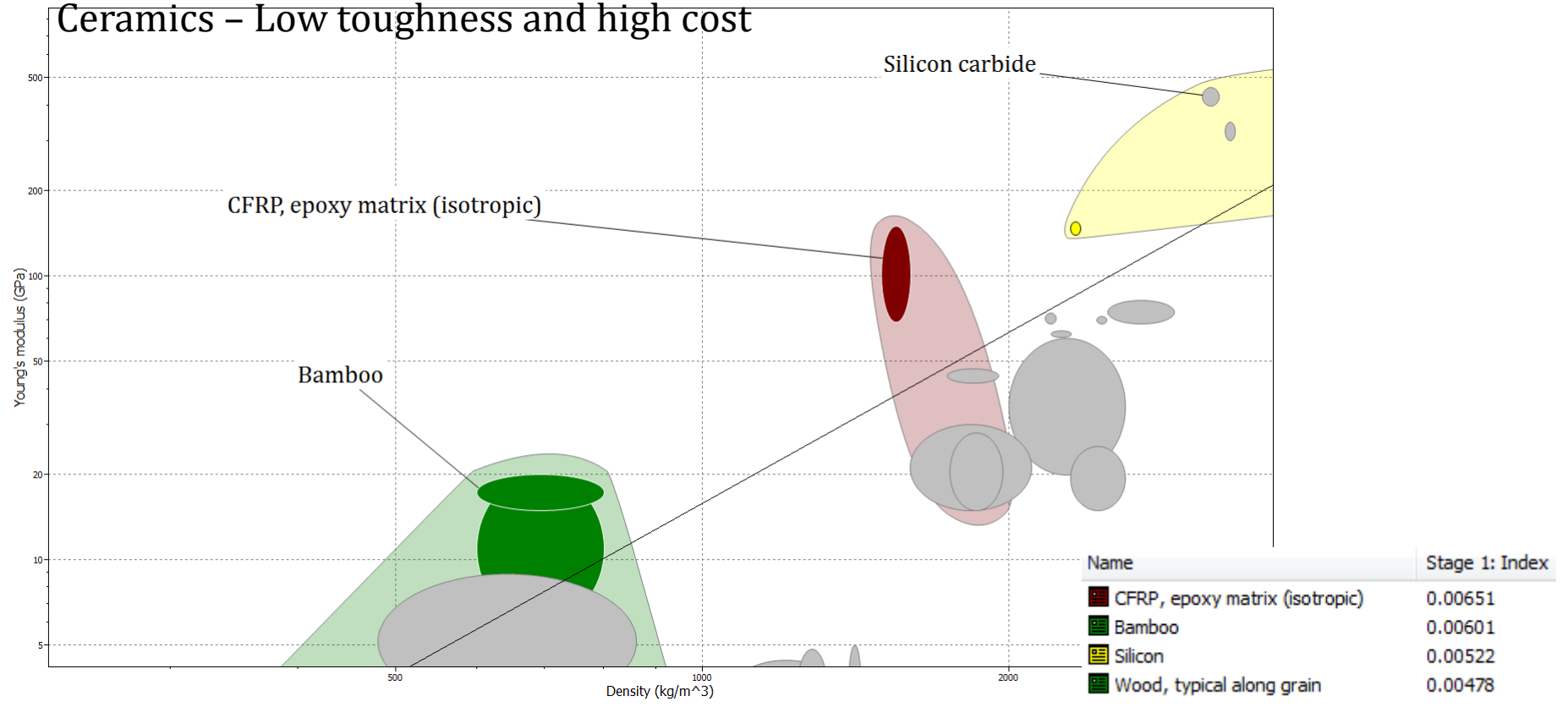


- CFRP - best material with more control of the properties
- Bamboo – Traditional material for oars for canoes
- Woods – Traditional, but with natural variabilities



$$10 \text{ GPa} < E < 200 \text{ GPa}$$

Ceramics – Low toughness and high cost





# Case Study 4: Materials for Light and Slender Oars

$$Solid I = \frac{\pi r^4}{4} = \frac{A^2}{4\pi}$$

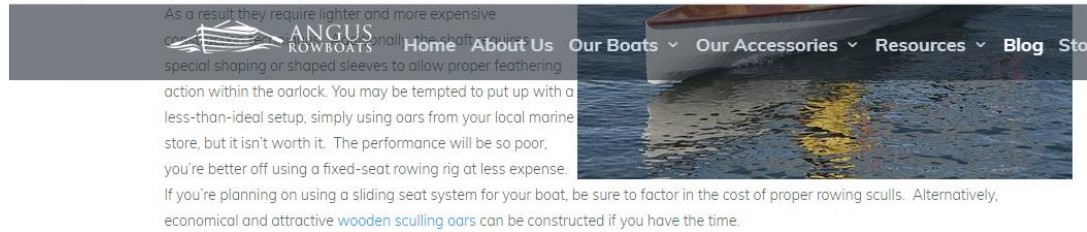
$$Tube I = \pi r^3 t$$

$$S \geq \frac{3 \cdot m^2}{4 \cdot \pi \cdot L^5} \cdot \frac{E}{\rho^2}$$

$m$  ↓ At fixed  $S_{min}$

$S_{min}$  ↑

$\delta_{max}$  ↓ At fixed  $m$



## OAR SPECS

Generally, sculling oars are 9' 6" in length, and construction is as light as possible. Carbon fiber oars weigh about 3.5 lbs each, while fiberglass and hollow shaft wood are about 4-5 lbs.

There are two main blade shapes – Macon and Hatchet (also known as cleaver). Macons are the traditional tulip-like shape and the oars are symmetrical on both sides, while Hatchets are asymmetrical with more blade extending down from the shaft into the water. Hatchets are either port or starboard. Both designs work well, however, hatchets are slightly more efficient. Macons on the other hand, are more effective if you decide to row without feathering since the blades are less likely to catch the water on the return stroke.



1,58 kg

Probably Tube shape

Assume 2,5 kg for a Solid Oar (exagerated)

CFRP ( $E = 110 \text{ GPa}$ ;  $\rho = 1550 \text{ kg/m}^3$ )  
Bamboo ( $E = 17,5 \text{ GPa}$ ;  $\rho = 700 \text{ kg/m}^3$ )

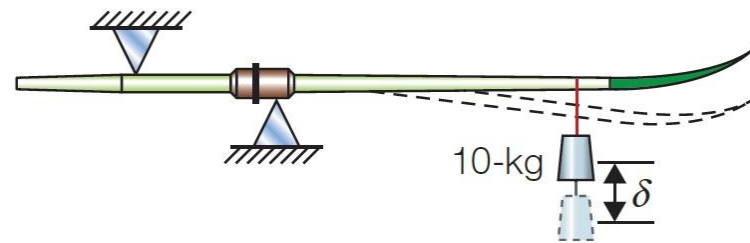
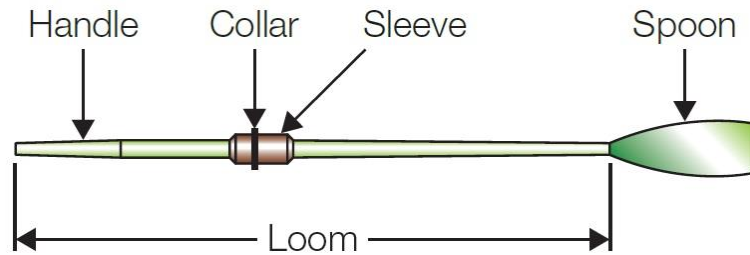
$S_{CFRP} = 853,94 \text{ N/m}$   
 $S_{Bamboo} = 666,1 \text{ N/m}$

CFRP good for Competition Oar





## Case Study 5: Materials for *CHEAP and Slender Oars*



$L$  (Outboard) = 2 m

Objective	<ul style="list-style-type: none"><li>Minimize the cost</li></ul>
Constraints	<ul style="list-style-type: none"><li>Stiffness specified</li><li>Length <math>L</math></li><li>Circular shape (beam)</li></ul>
Free Variables	<ul style="list-style-type: none"><li>Area (<math>A</math>) of the cross-section</li><li>Choice of the material</li></ul>



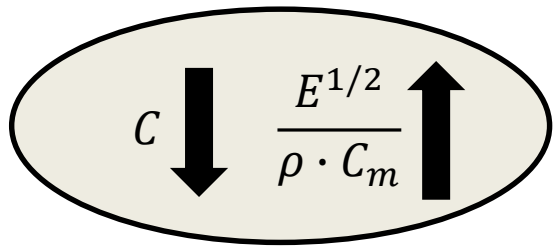


## Case Study 5: Materials for CHEAP and Slender Oars

$$\left\{ \begin{array}{l} m \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho}{E^{1/2}} \\ C = m \cdot C_m \end{array} \right. \longrightarrow m = \frac{C}{C_m}$$

$C$  Cost  
 $C_m$  Cost per unit of mass  
↓  
Better to consider cost always  
as a function of mass

$$C \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho \cdot C_m}{E^{1/2}}$$





# Case Study 5: Materials for CHEAP and Slender Oars

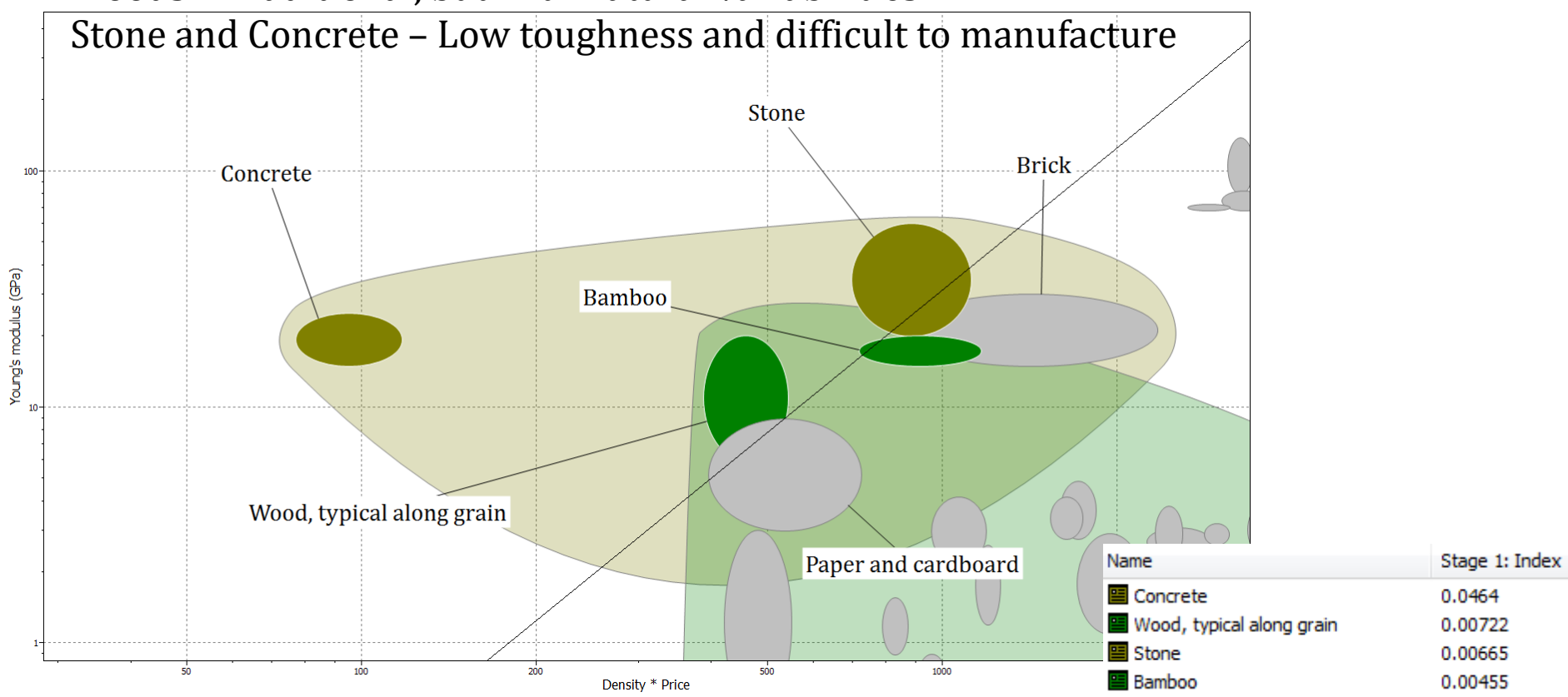
$$C \downarrow \quad \frac{E^{1/2}}{\rho \cdot C_m} \uparrow$$



$$10 \text{ Gpa} < E < 200 \text{ GPa}$$

Bamboo – Traditional material for oars for canoes  
 Woods – Traditional, but with natural variabilities

Stone and Concrete – Low toughness and difficult to manufacture



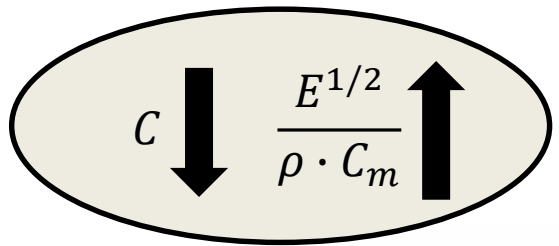


## Case Study 5: Materials for CHEAP and Slender Oars

$$\left\{ \begin{array}{l} m \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho}{E^{1/2}} \\ C = m \cdot C_m \end{array} \right. \longrightarrow m = \frac{C}{C_m}$$

$$C \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho \cdot C_m}{E^{1/2}}$$

$C$  Cost  
 $C_m$  Cost per unit of mass  
↓  
Better to consider cost always  
as a function of mass



Woods good for Commercial Oar





# The Strength design

**The Strength design is important to avoid plastic collapse... or maybe not**

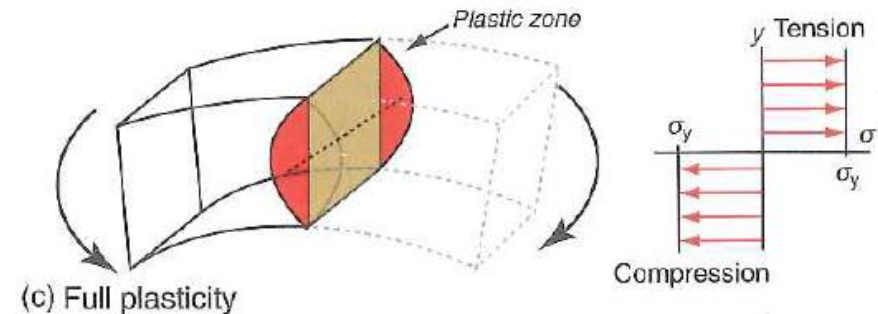
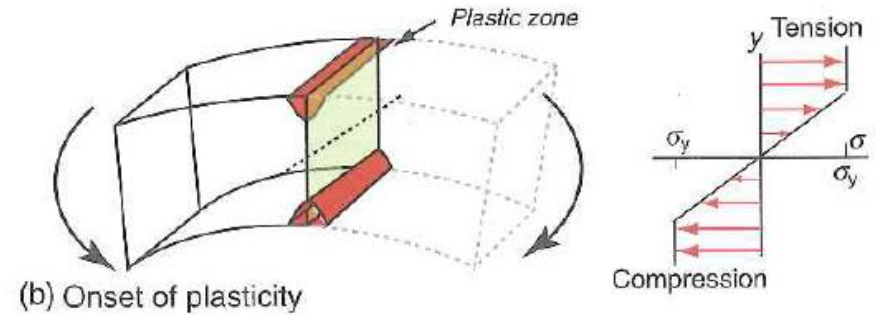
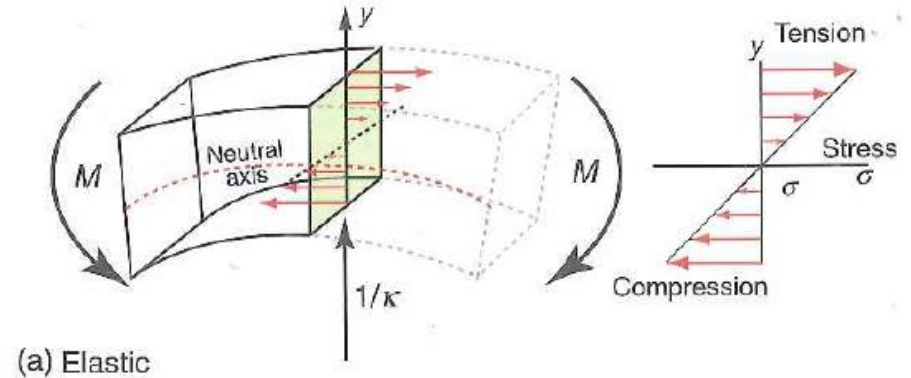
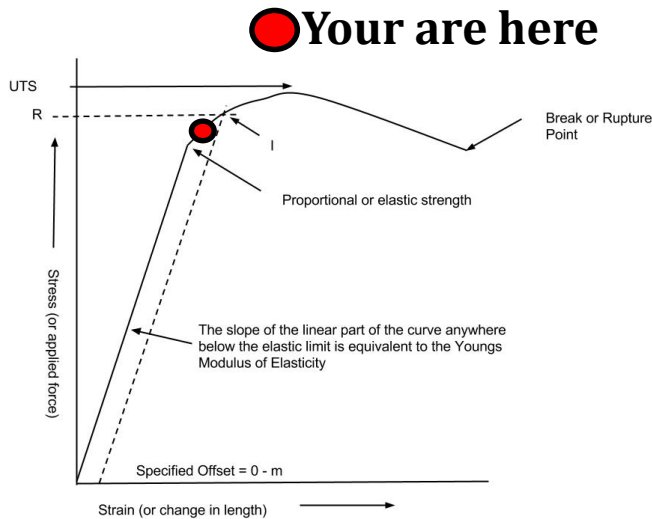






# The Strength design

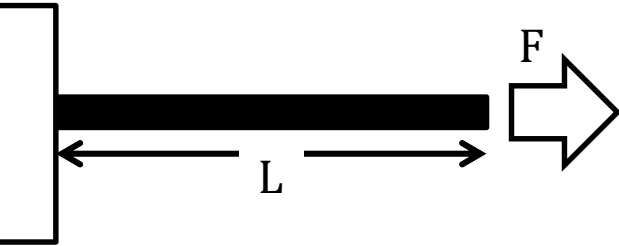
The Strength design is important to avoid plastic collapse





# Lightest Tie-Rod (Traction conditions)

**Case Study 6:**  
**Find the Lightest *STRONG* Tie-Rod**



Objective	<ul style="list-style-type: none"> <li>Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Support tensile load F without yielding</li> <li>Length L</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Area (A) of the cross-section</li> <li>Choice of the material</li> </ul>

## DATA

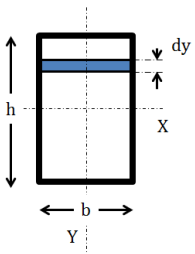
F = 1000 N

## Dimensions:

Length: 300 mm

Thickness = 1 mm

Width = 25 mm



**In Traction,  
the shape of the cross-section is not important**

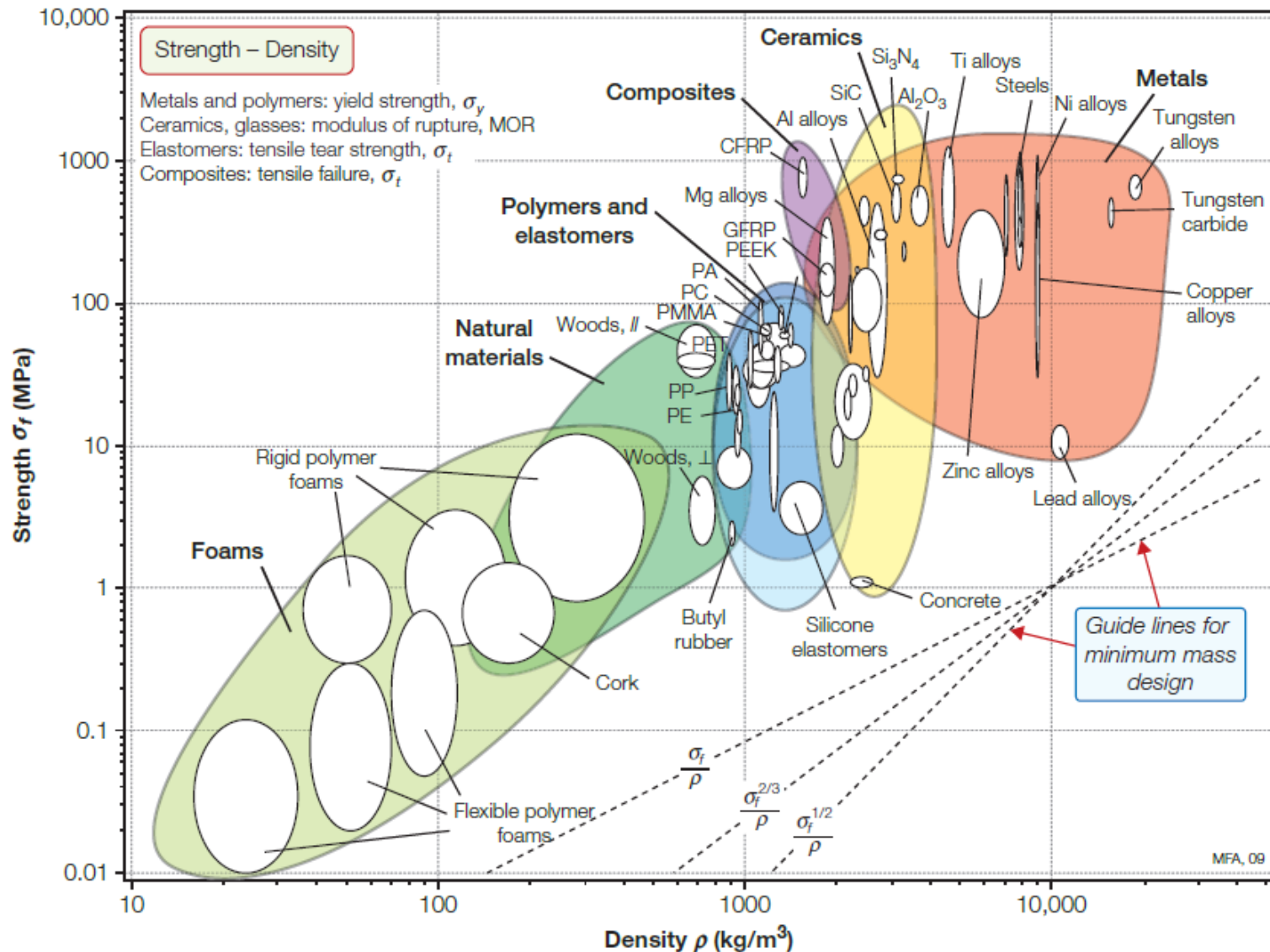
$$m = A \cdot L \cdot \rho \longrightarrow A = \frac{m}{L \cdot \rho}$$

From material :  $\frac{F}{A} \leq \sigma_y$

$$\longrightarrow m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}$$



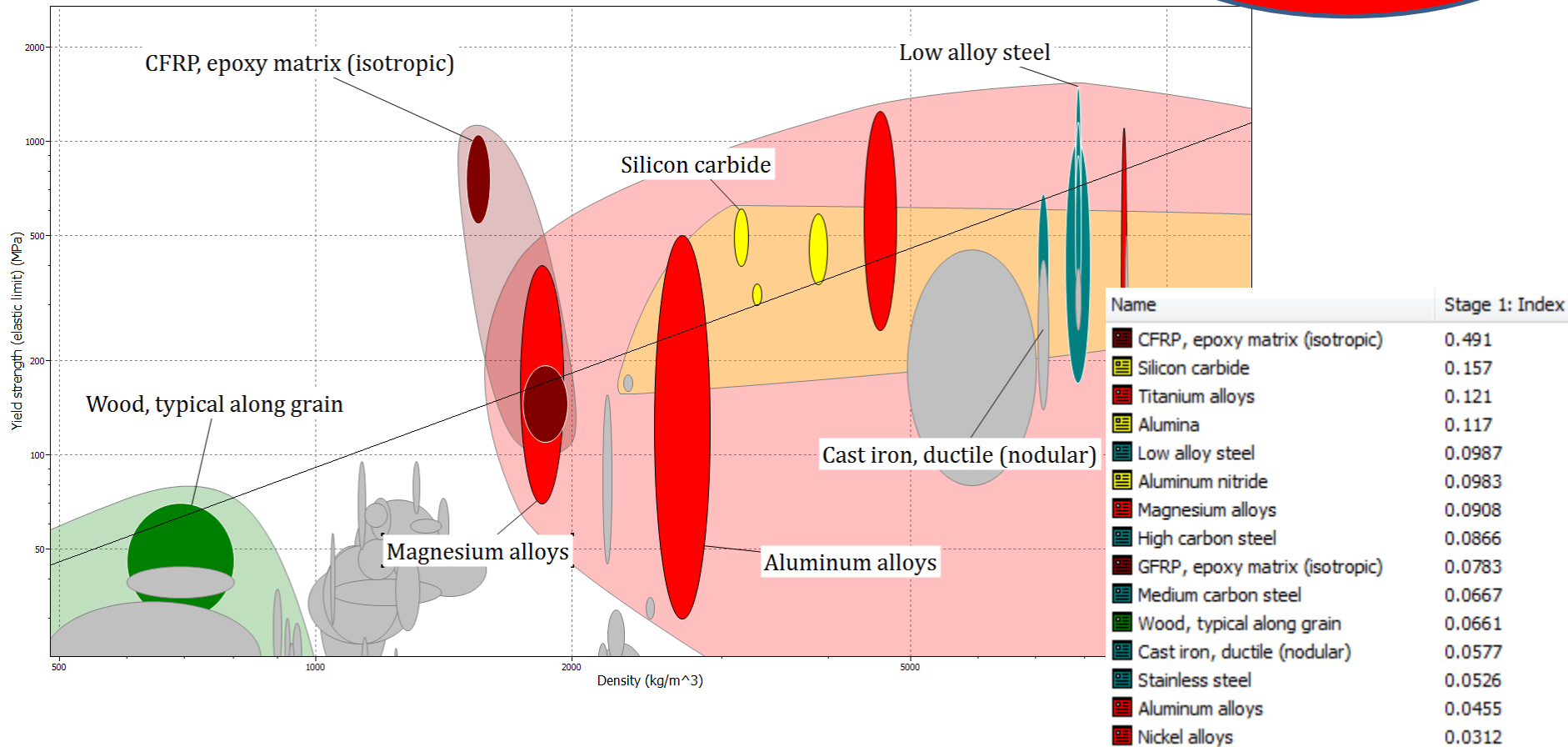
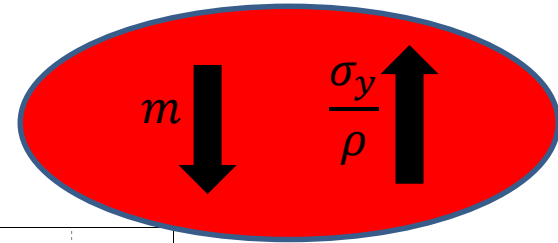
# Ashby Diagrams





# CES

**Case Study 6:**  
**Find the Lightest STRONG Tie-Rod**

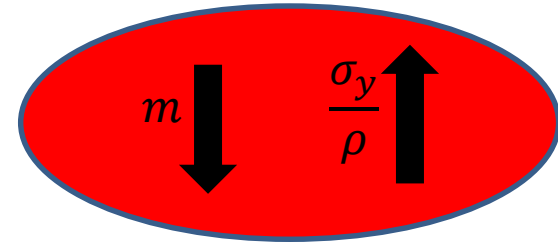




# Lightest Tie-Rod (Traction conditions)

**Case Study 6:**  
**Find the Lightest *STRONG* Tie-Rod**

$$m \geq F \cdot L \cdot \frac{\rho}{\sigma_y}$$



It is possible to do as before, but let's calculate the maximum F on the precedent Tie-Rod

Material	Weight (kg)	Width and Thickness (mm)
Al Alloys	1,25	63

- Stainless Steel ( $\sigma_y = 600 \text{ MPa}$ ;  $\rho = 7800 \text{ kg/m}^3$ )
- Wood ( $\sigma_y = 50 \text{ MPa}$ ;  $\rho = 700 \text{ kg/m}^3$ )
- Al Alloys ( $\sigma_y = 270 \text{ MPa}$ ;  $\rho = 75 \text{ kg/m}^3$ )

0 kN  $F \leq \frac{m}{L} \cdot \frac{\sigma_y}{\rho} = 416 \text{ kN}$  X kN

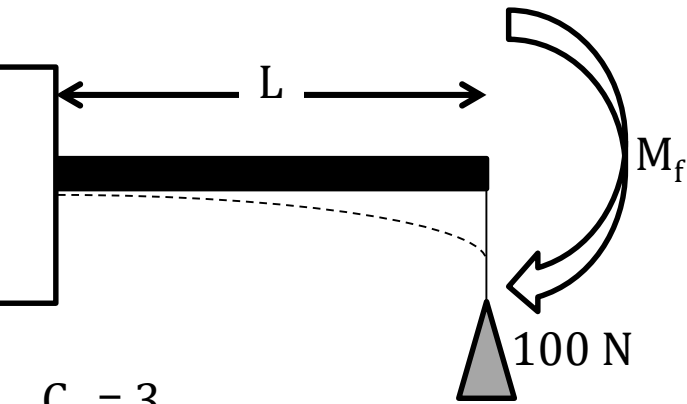
Elastic Throughout Plastic deformation/ Collapse





# Lightest Beam (Bending conditions)

**Case Study 7:**  
**Find the Lightest STRONG Beam**



$$C_1 = 3$$

Length: 300 mm

Hypothesis:

- $y_{max} = h/2$
- $\sigma_{max} \geq \sigma$

$$\left\{ \begin{array}{l} \sigma_{max} = \frac{M_f \cdot y_{max}}{I} \leq \sigma_f \\ m = A \cdot L \cdot \rho \end{array} \right. \rightarrow A = \frac{m}{L \cdot \rho}$$

M = Moment

$\sigma$  = Stress

$y_{max}$  = max distance from the neutral axis  $\rightarrow \sigma_{max}$

Objective	<ul style="list-style-type: none"> <li>• Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>• Stiffness specified</li> <li>• Length L</li> <li>• Square shape</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>• Area (A) of the cross-section</li> <li>• Choice of the material</li> </ul>

**Beam: Square Section**

$$b = h$$

Since  $A = b^2$

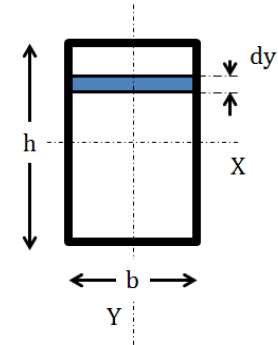
$$I = \frac{bh^3}{12} \rightarrow \frac{A^{3/2}}{6}$$



# Lightest Beam (Bending conditions)

**Case Study 7:**  
**Find the Lightest STRONG Beam**

$$I' = \frac{bh^2}{6} \rightarrow \frac{A^{3/2}}{6}$$

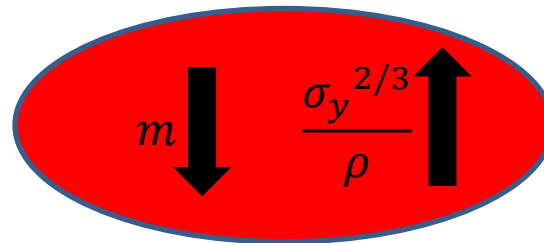


$$\sigma_{max} = \frac{M_f \cdot y_{max}}{I} = \frac{M_f \cdot \frac{h}{2}}{\frac{bh^3}{12}} = \frac{M_f}{I'} \leq \sigma_f$$

$m = A \cdot L \cdot \rho \rightarrow A = \frac{m}{L \cdot \rho}$

$$\sigma_f \geq \frac{M_f \cdot 6}{A^{3/2}} = \frac{M_f \cdot 6 \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}}$$

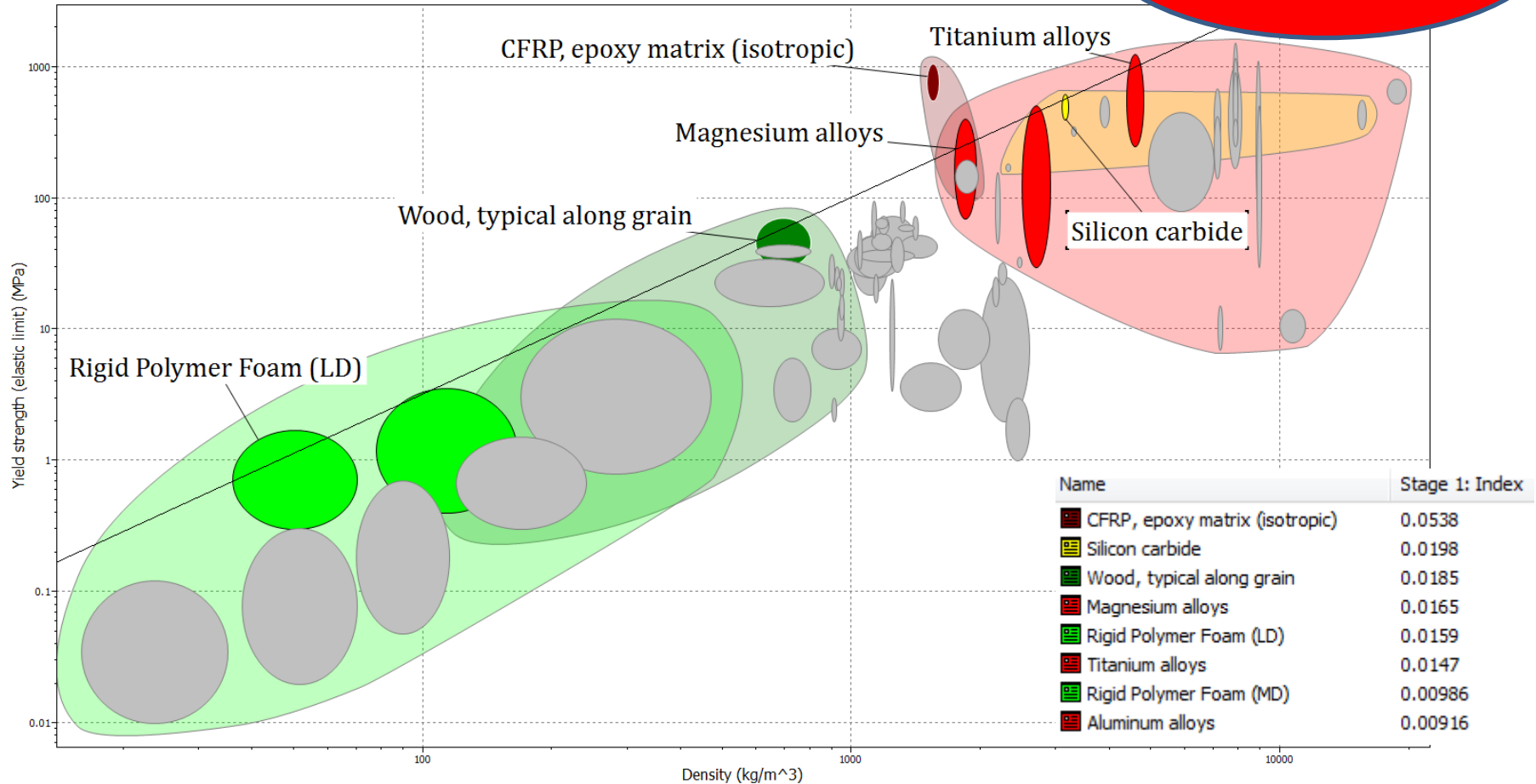
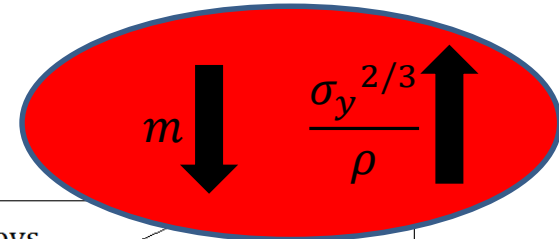
$$\downarrow$$
$$m \geq (M_f \cdot 6)^{2/3} \cdot L \cdot \frac{\rho}{\sigma_f^{2/3}}$$





# CES

## Case Study 7: Find the Lightest STRONG Beam



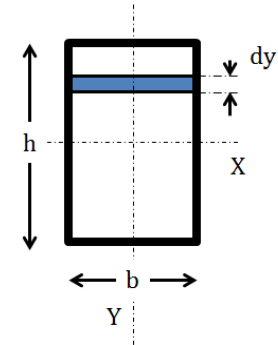




# Lightest Beam (Bending conditions)

**Case Study 7:**  
**Find the Lightest STRONG Beam**

$$I' = \frac{bh^2}{6} \gg \frac{A^{3/2}}{6}$$



$$\sigma_{max} = \frac{M_f \cdot y_{max}}{I} = \frac{M_f \cdot \frac{h}{2}}{\frac{bh^3}{12}} = \frac{M_f}{I'} \leq \sigma_f$$

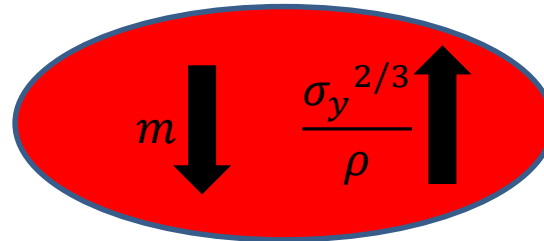
$$m = A \cdot L \cdot \rho \rightarrow A = \frac{m}{L \cdot \rho}$$

$$\sigma_f \geq \frac{M_f \cdot 6}{A^{3/2}} = \frac{M_f \cdot 6 \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}}$$



$$m \geq (M_f \cdot 6)^{2/3} \cdot L \cdot \frac{\rho}{\sigma_f^{2/3}}$$

- It is always better to choose a shape that uses less material to provide the same strength TO SUPPORT BENDING

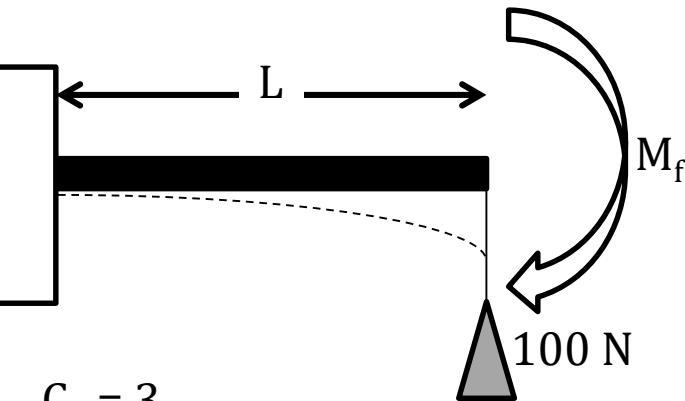




# Lightest Panel (Bending conditions)

**Case Study 8:**  
**Find the Lightest STRONG Panel**

Objective	<ul style="list-style-type: none"> <li>Minimize the mass</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Stiffness specified</li> <li>Length L and b specified</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>h (thickness) of the cross-section</li> <li>Choice of the material</li> </ul>

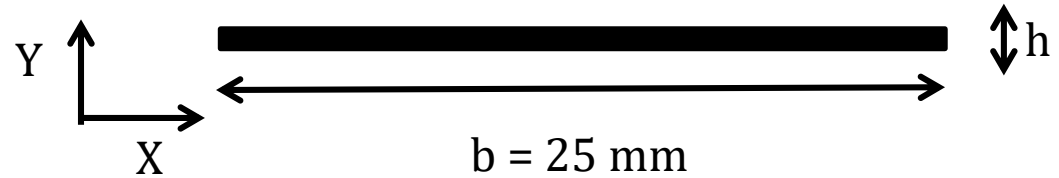


$C_1 = 3$

Length: 300 mm

Hypothesis:

- $y_{max} = h/2$
- $\sigma_{max} \geq \sigma$



$$\left\{ \begin{array}{l} \sigma_{max} = \frac{M_f \cdot y_{max}}{I} \leq \sigma_f \\ m = A \cdot L \cdot \rho \end{array} \right. \rightarrow A = \frac{m}{L \cdot \rho}$$

$$I' = \frac{bh^2}{6} \gg \frac{A^2}{b \cdot 6}$$

Since  $A = bh$

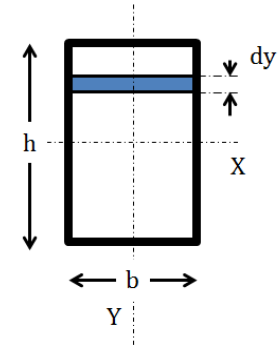
$$h = \frac{A}{b}$$



# Lightest Panel (Bending conditions)

**Case Study 8:**  
**Find the Lightest STRONG Panel**

$$I' = \frac{bh^2}{6} \gg \frac{A^2}{b \cdot 6}$$



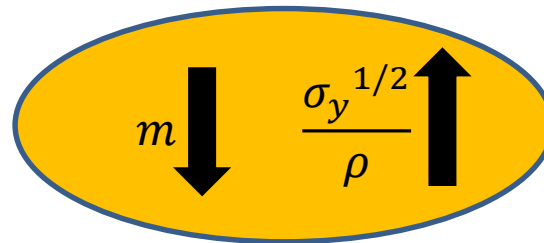
$$\sigma_{max} = \frac{M_f \cdot y_{max}}{I} = \frac{M_f \cdot b \cdot 6}{A^2} = \frac{M_f}{I'} \leq \sigma_f$$

$m = A \cdot L \cdot \rho \rightarrow A = \frac{m}{L \cdot \rho}$

$$\sigma_f \geq \frac{M_f \cdot b \cdot 6}{A^2} = \frac{M_f \cdot 6 \cdot b \cdot L^2 \cdot \rho^2}{m^2}$$

$$\downarrow$$

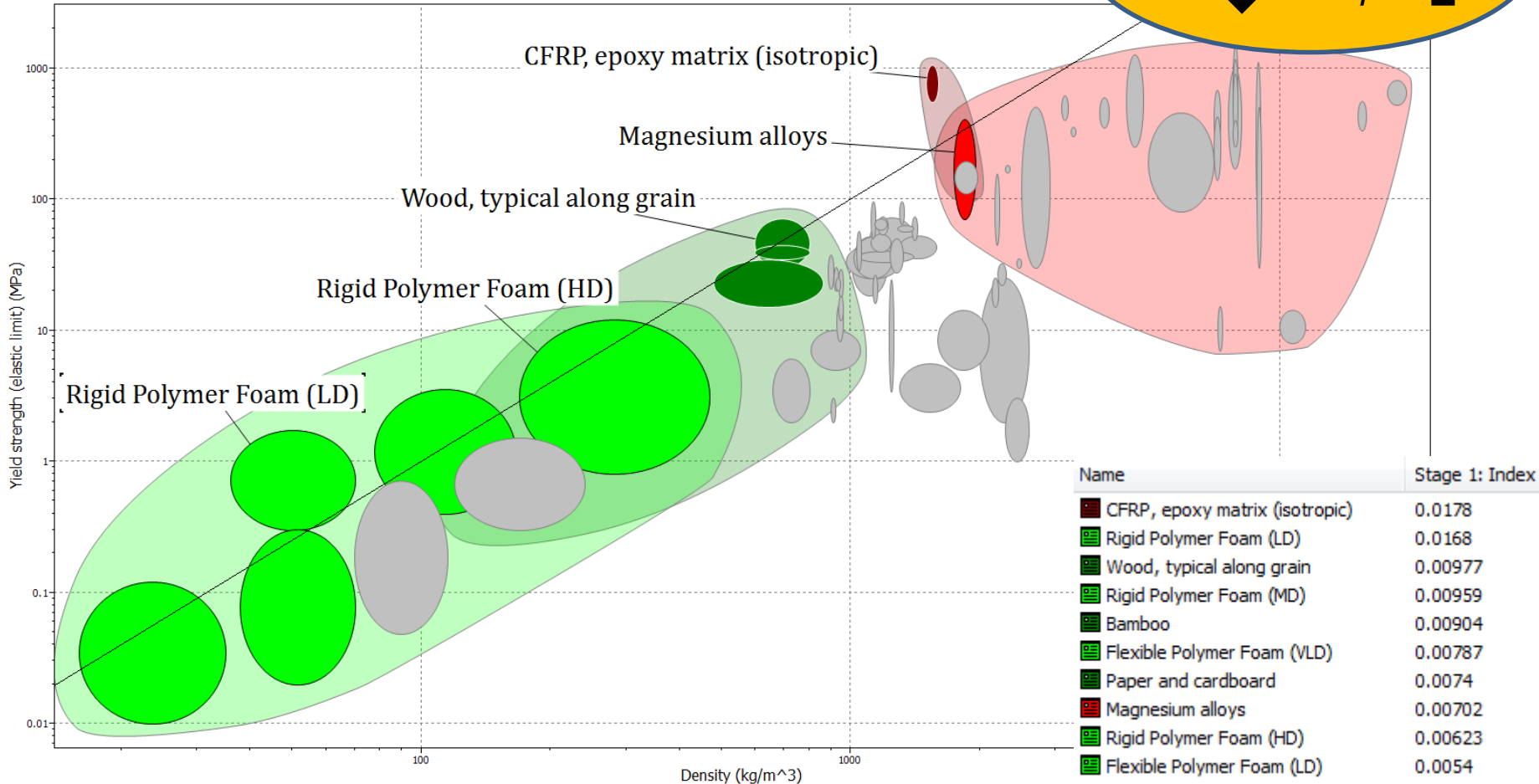
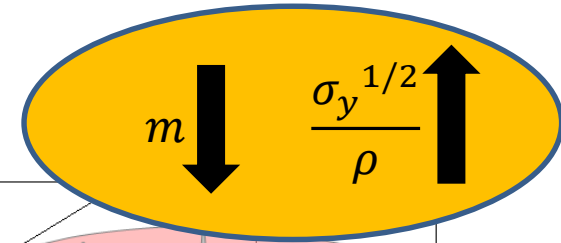
$$m \geq (M_f \cdot 6 \cdot b)^{1/2} \cdot L \cdot \frac{\rho}{\sigma_f^{1/2}}$$





# CES

## Case Study 8: Find the Lightest STRONG Panel

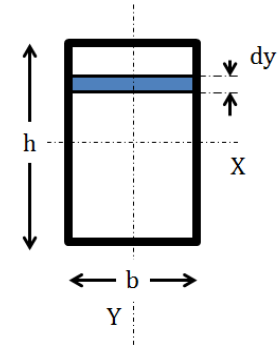




# Lightest Panel (Bending conditions)

**Case Study 8:**  
**Find the Lightest STRONG Panel**

$$I' = \frac{bh^2}{6} \gg \frac{A^{3/2}}{6}$$



$$\sigma_{max} = \frac{M_f \cdot y_{max}}{I} = \frac{M_f \cdot \frac{h}{2}}{\frac{bh^3}{12}} = \frac{M_f}{I'} \leq \sigma_f$$

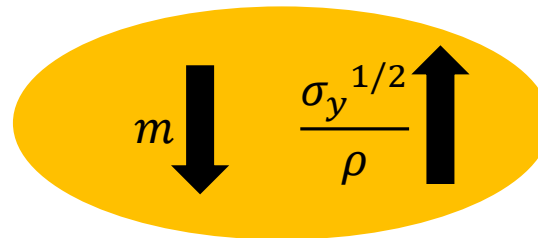
$m = A \cdot L \cdot \rho \rightarrow A = \frac{m}{L \cdot \rho}$

$$\sigma_f \geq \frac{M_f \cdot 6}{A^{3/2}} = \frac{M_f \cdot 6 \cdot L^{3/2} \cdot \rho^{3/2}}{m^{3/2}}$$



$$m \geq (M_f \cdot 6 \cdot b)^{1/2} \cdot L \cdot \frac{\rho}{\sigma_f^{1/2}}$$

- It is always better to choose a shape that uses less material to provide the same strength TO SUPPORT BENDING





# Summary (to minimize the mass)

## Stiffness – Traction :

Name	Stage 1: Index
Silicon carbide	0.136
Aluminum nitride	0.0984
Alumina	0.094
CFRP, epoxy matrix (isotropic)	0.0657
Silicon	0.0634
Tungsten carbides	0.0425
Silica glass	0.0323
Soda-lime glass	0.0284
Borosilicate glass	0.0278
Aluminum alloys	0.0277
Bamboo	0.025
Wood, typical along grain	0.0158

## Stiffness – Bending (Beam):

Name	Stage 1: Index
Silicon carbide	0.00657
CFRP, epoxy matrix (isotropic)	0.00651
Bamboo	0.00601
Aluminum nitride	0.00546
Silicon	0.00522
Alumina	0.00492
Wood, typical along grain	0.00478
Rigid Polymer Foam (LD)	0.00413
Silica glass	0.00384
Magnesium alloys	0.00362
Paper and cardboard	0.00354
Rigid Polymer Foam (MD)	0.00314

## Stiffness – Bending (Panel):

Name	Stage 1: Index
Rigid Polymer Foam (LD)	0.00697
Rigid Polymer Foam (MD)	0.00442
Bamboo	0.00373
Flexible Polymer Foam (VLD)	0.00335
Wood, typical along grain	0.00321
CFRP, epoxy matrix (isotropic)	0.00301
Paper and cardboard	0.00269
Rigid Polymer Foam (HD)	0.00239
Flexible Polymer Foam (LD)	0.00233
Flexible Polymer Foam (MD)	0.00212
Cork	0.00173

## Strength – Traction :

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.491
Silicon carbide	0.157
Titanium alloys	0.121
Alumina	0.117
Low alloy steel	0.0987
Aluminum nitride	0.0983
Magnesium alloys	0.0908
High carbon steel	0.0866
GFRP, epoxy matrix (isotropic)	0.0783
Medium carbon steel	0.0667
Wood, typical along grain	0.0661
Cast iron, ductile (nodular)	0.0577
Stainless steel	0.0526
Aluminum alloys	0.0455
Nickel alloys	0.0312

$\frac{E}{\rho}$	$\frac{E^{1/2}}{\rho}$	$\frac{E^{1/3}}{\rho}$
$\frac{\sigma_y}{\rho}$	$\frac{\sigma_y^{2/3}}{\rho}$	$\frac{\sigma_y^{1/2}}{\rho}$

## Strength – Bending :

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.0538
Silicon carbide	0.0198
Wood, typical along grain	0.0185
Magnesium alloys	0.0165
Rigid Polymer Foam (LD)	0.0159
Titanium alloys	0.0147
Rigid Polymer Foam (MD)	0.00986
Aluminum alloys	0.00916

## Strength – Bending (Panel):

Name	Stage 1: Index
CFRP, epoxy matrix (isotropic)	0.0178
Rigid Polymer Foam (LD)	0.0168
Wood, typical along grain	0.00977
Rigid Polymer Foam (MD)	0.00959
Bamboo	0.00904
Flexible Polymer Foam (VLD)	0.00787
Paper and cardboard	0.0074
Magnesium alloys	0.00702
Rigid Polymer Foam (HD)	0.00623
Flexible Polymer Foam (LD)	0.0054



Some data : Nowadays, half the expense of building a house is the cost of the materials

Family house : 200 tons

Large apartment block : 20,000 tons

***Case Study 9:  
Materials for Constructions***





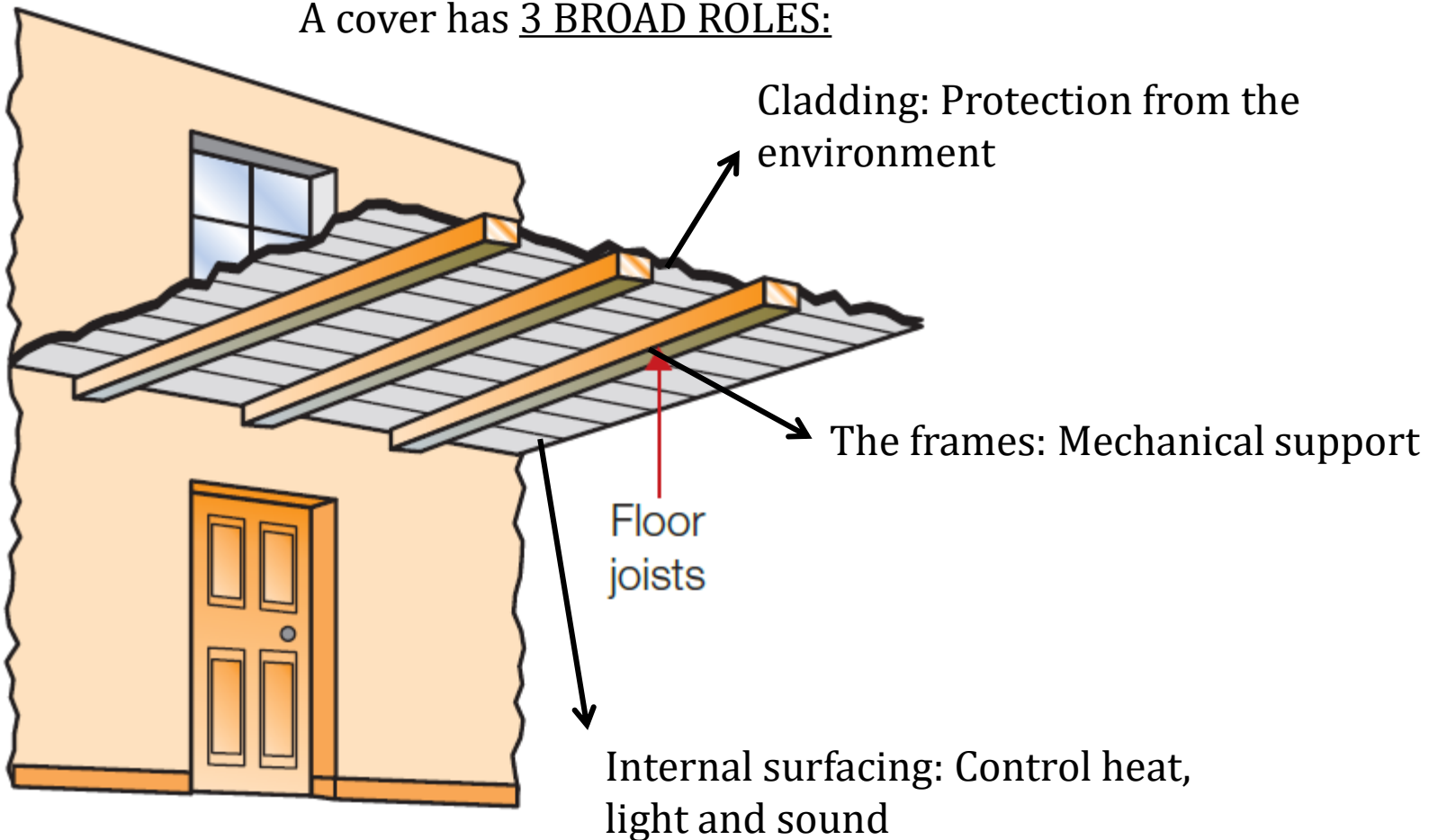
Mr. Pincopallo asks  
a new cover



Understand the problem and  
translate it in selection criteria,  
thus properties,

## Case Study 9: Materials for Constructions

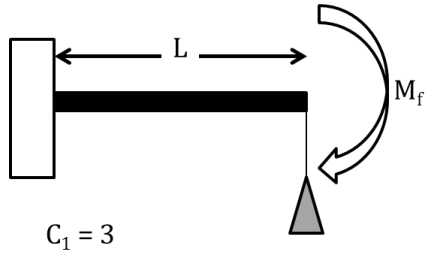
A cover has 3 BROAD ROLES:







**Case Study 9:  
Materials for Constructions  
(Structural Frame)**



Objective	<ul style="list-style-type: none"> <li>Minimize the cost</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Length L specified</li> <li>Stiffness: must not deflect too much under loads</li> <li>Strength: must not fall under design loads</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Area (A) of the cross-section</li> <li>Choice of the material</li> </ul>

Hypothesis:

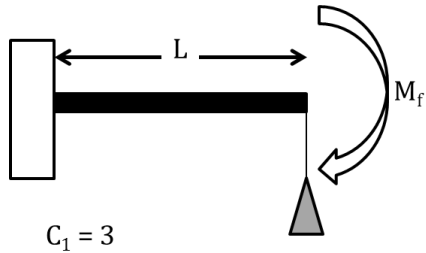
- $\frac{F}{\delta} \geq S_{min} = S$

**Floor joints are beams, loaded in bending.**

$$\left\{ \begin{array}{l} \frac{F}{\delta} \geq S_{min} = \frac{C_1 EI}{L^3} \\ m = A \cdot L \cdot \rho \longrightarrow A = \frac{m}{L \cdot \rho} \\ C = m \cdot C_m \longrightarrow m = \frac{C}{C_m} \end{array} \right. \longrightarrow C \geq \left( \frac{4 \cdot \pi \cdot S \cdot L^5}{3} \right)^{1/2} \cdot \frac{\rho \cdot C_m}{E^{1/2}}$$



**Case Study 9:  
Materials for Constructions  
(Structural Frame)**



Objective	<ul style="list-style-type: none"> <li>Minimize the cost</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Length L specified</li> <li>Stiffness: must not deflect too much under loads</li> <li>Strength: must not fall under design loads</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Area (A) of the cross-section</li> <li>Choice of the material</li> </ul>

$$I = \frac{bh^3}{12} \gg \frac{A^{3/2}}{6}$$

**Floor joints are beams, loaded in bending.**

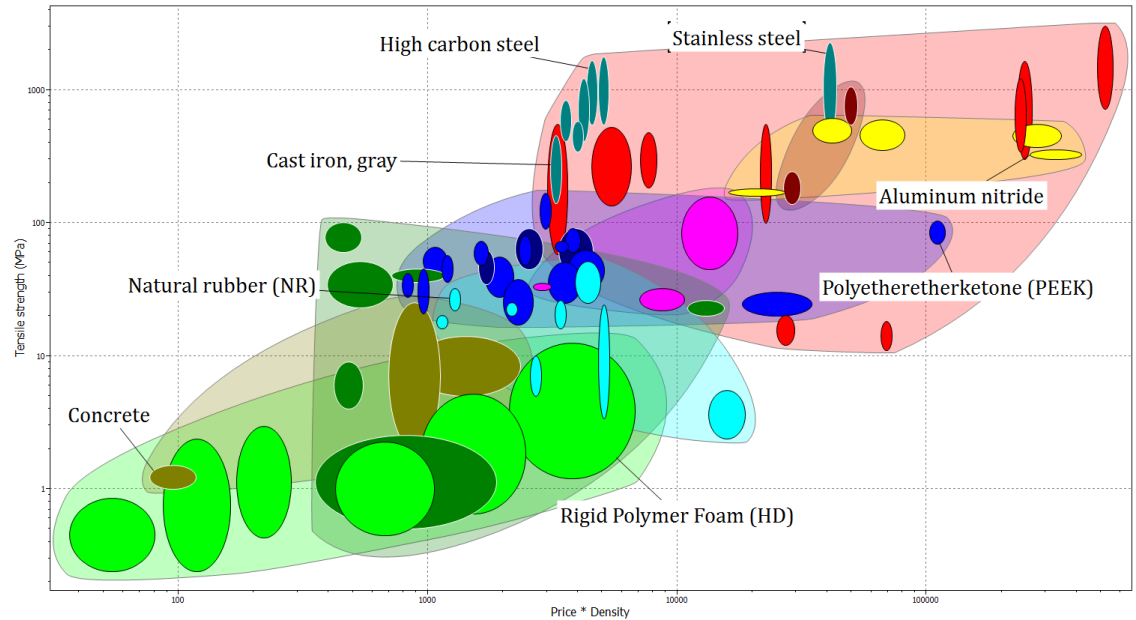
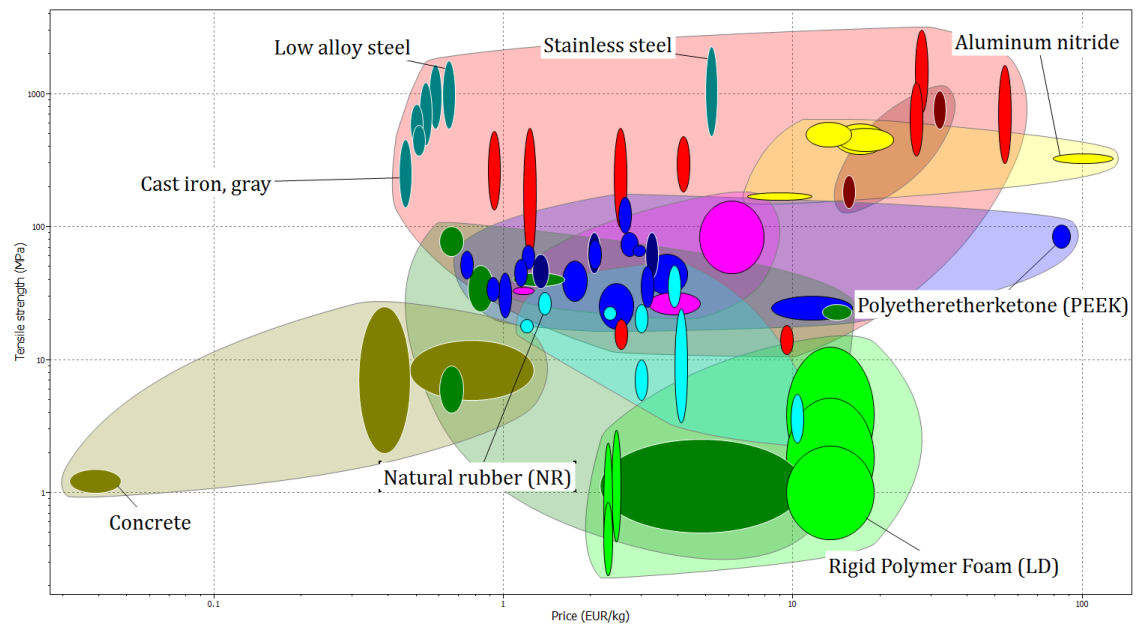
$$\left\{ \begin{array}{l} \sigma_{max} = \frac{M_f \cdot y_{max}}{I} \leq \sigma_f \\ m = A \cdot L \cdot \rho \longrightarrow A = \frac{m}{L \cdot \rho} \\ C = m \cdot C_m \longrightarrow m = \frac{C}{C_m} \end{array} \right. \longrightarrow C \geq (M_f \cdot 6)^{2/3} \cdot L \cdot \frac{\rho \cdot C_m}{\sigma_f^{2/3}}$$



## Case Study 9: Materials for Constructions

ATTENTION!!!

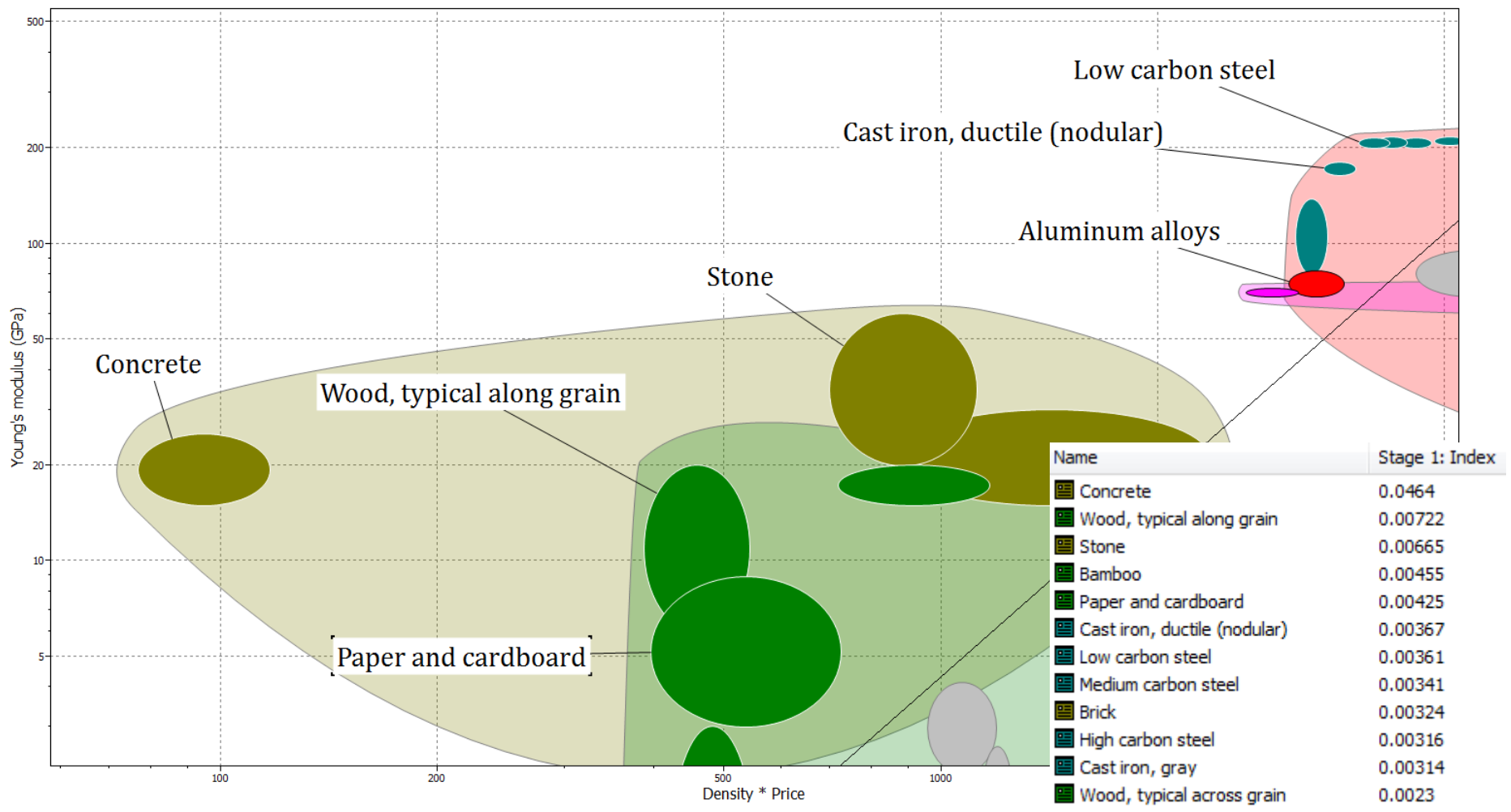
Selection with the cost/kg and  
with the cost/m<sup>3</sup> is  
DIFFERENT





# Case Study 9: Materials for Constructions

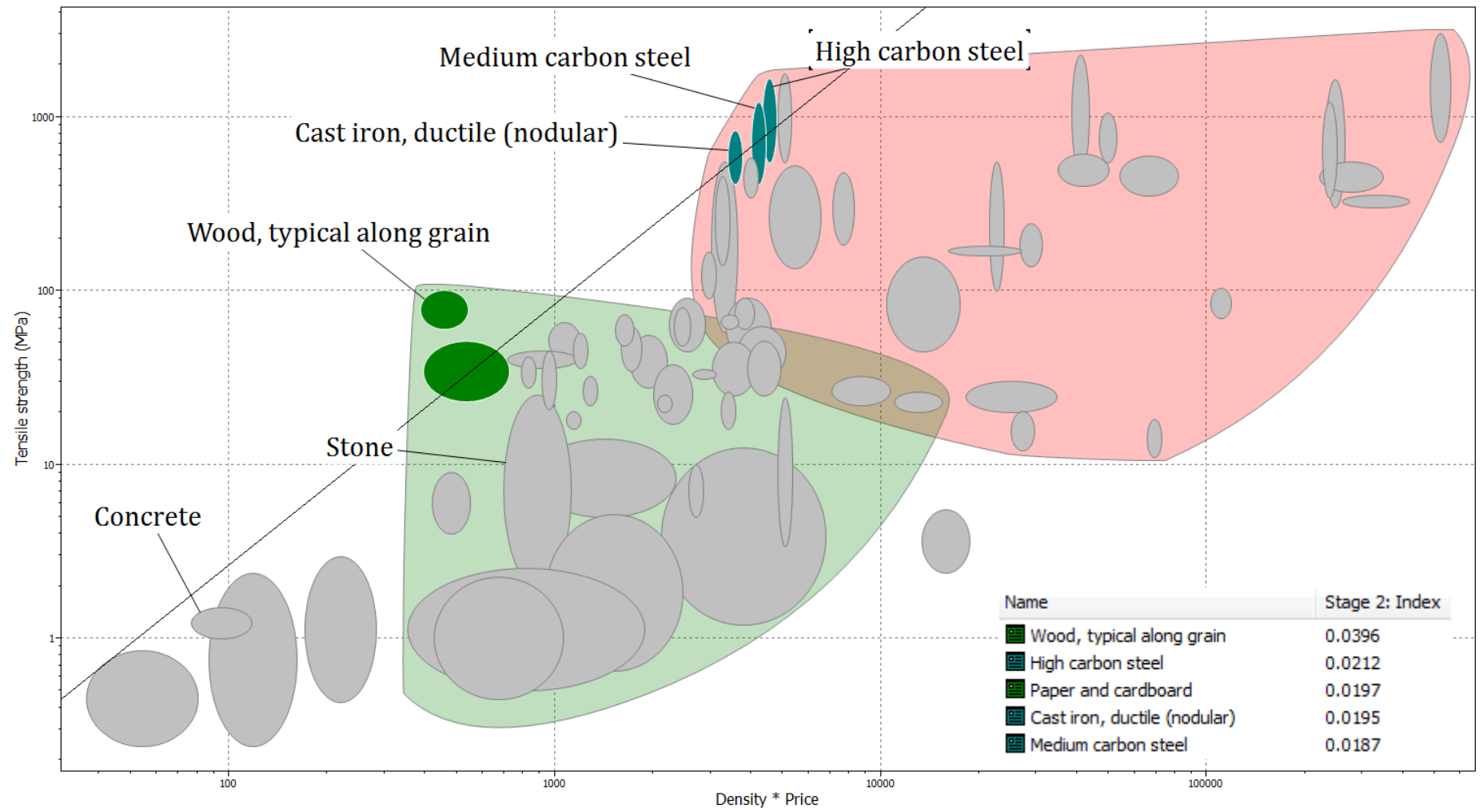
$$C \downarrow \frac{E^{1/2}}{\rho \cdot C_m} \uparrow$$





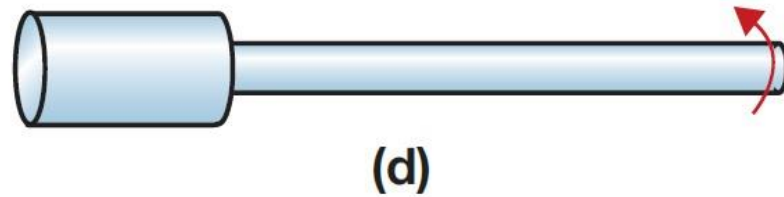
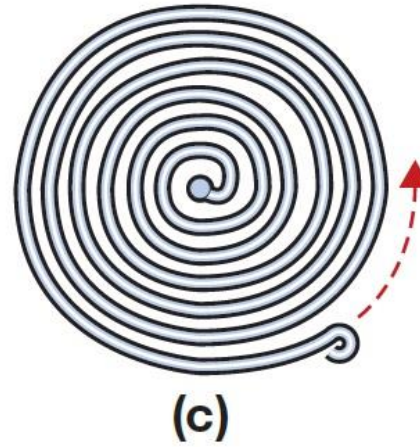
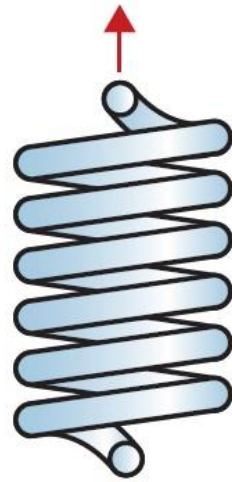
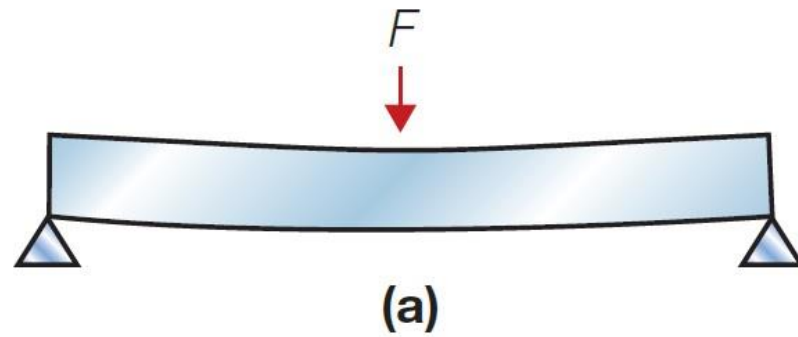
# Case Study 9: Materials for Constructions

$$C \downarrow \frac{\sigma_f^{2/3}}{\rho \cdot C_m} \uparrow$$





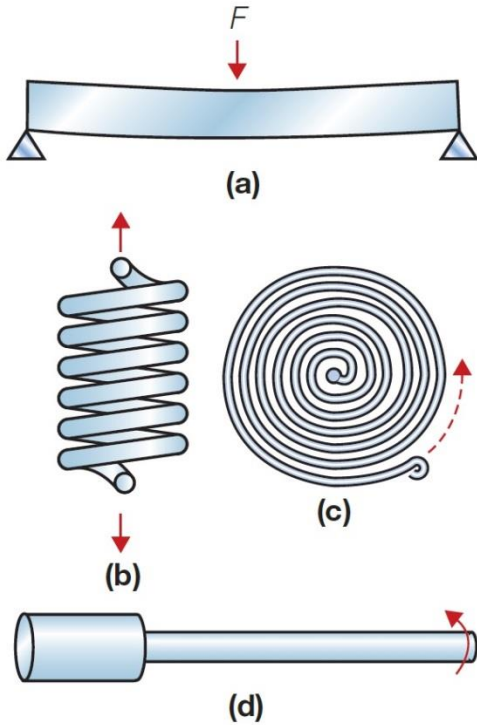
**Case Study :  
Materials for Springs**





**Case Study 10:  
Materials for Small Springs**

Objective	<ul style="list-style-type: none"> <li>Maximize stored elastic energy</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>No failure <math>\rightarrow \sigma &lt; \sigma_f</math> throughout the spring</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Choice of the material</li> </ul>

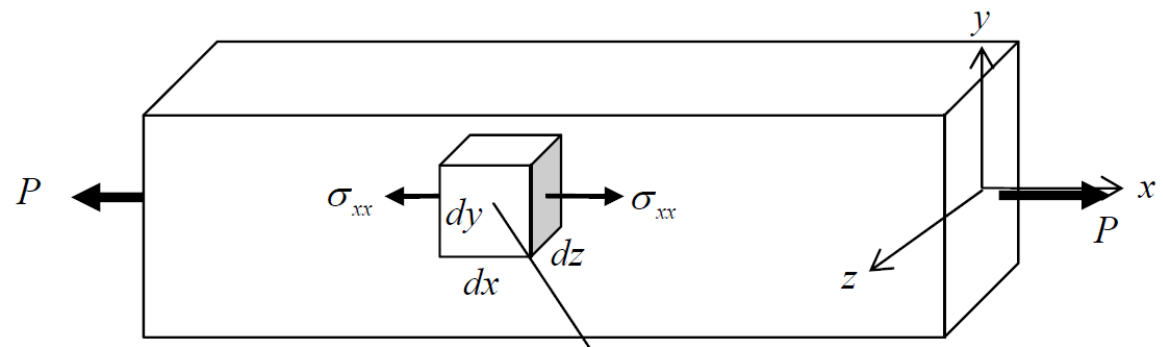


*Condition of elasticity*

$$\left\{ \begin{array}{l} \sigma_y \geq \sigma \\ \sigma = E \cdot \varepsilon \rightarrow \varepsilon = \frac{\sigma}{E} \end{array} \right.$$

**SMALL??  $\rightarrow$  V FREE VARIABLE!!**

$$dV = dx dy dz$$



[Solid Mechanics Part I Kelly]

volume element



**Case Study 10:  
Materials for Small Springs**

$$\left\{ \begin{array}{l} \sigma_y \geq \sigma \\ \varepsilon = \frac{\sigma}{E} \end{array} \right.$$

$$dV = dx dy dz$$

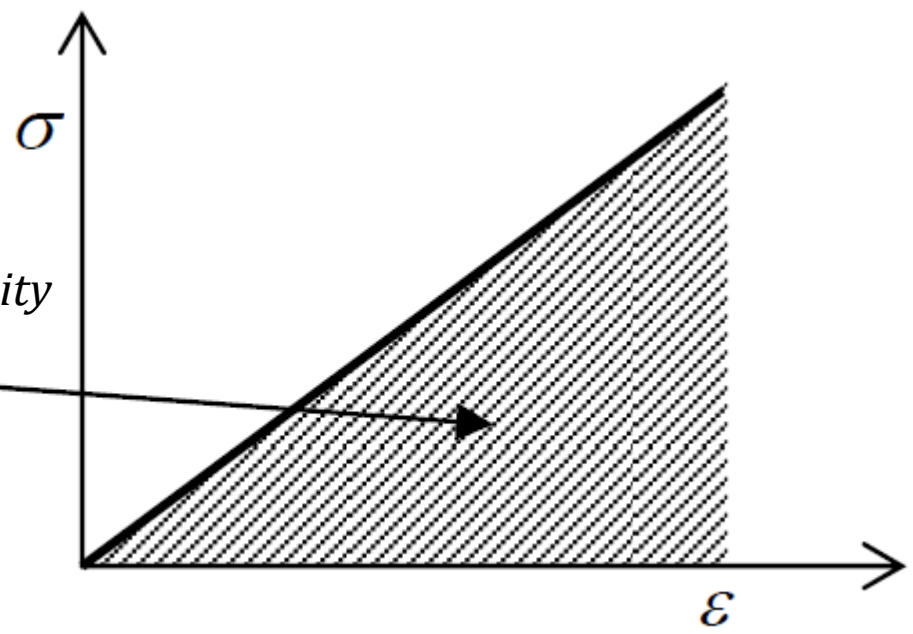
$$\left\{ \begin{array}{l} m = V \cdot \rho \\ W_{el} = \frac{1}{2} \int \sigma \cdot \varepsilon dV = \frac{1}{2} \sigma \cdot \varepsilon \cdot V \end{array} \right.$$

*Total strain energy in the piece considered*

$$U = \frac{(\sigma_{xx} dy dz)^2 dx}{2E dy dz}$$

*Strain energy density*

*u*



$$W_{el} = \frac{\sigma_y^2}{2E} = M_1$$

*Total strain energy  
PER UNIT OF VOLUME*

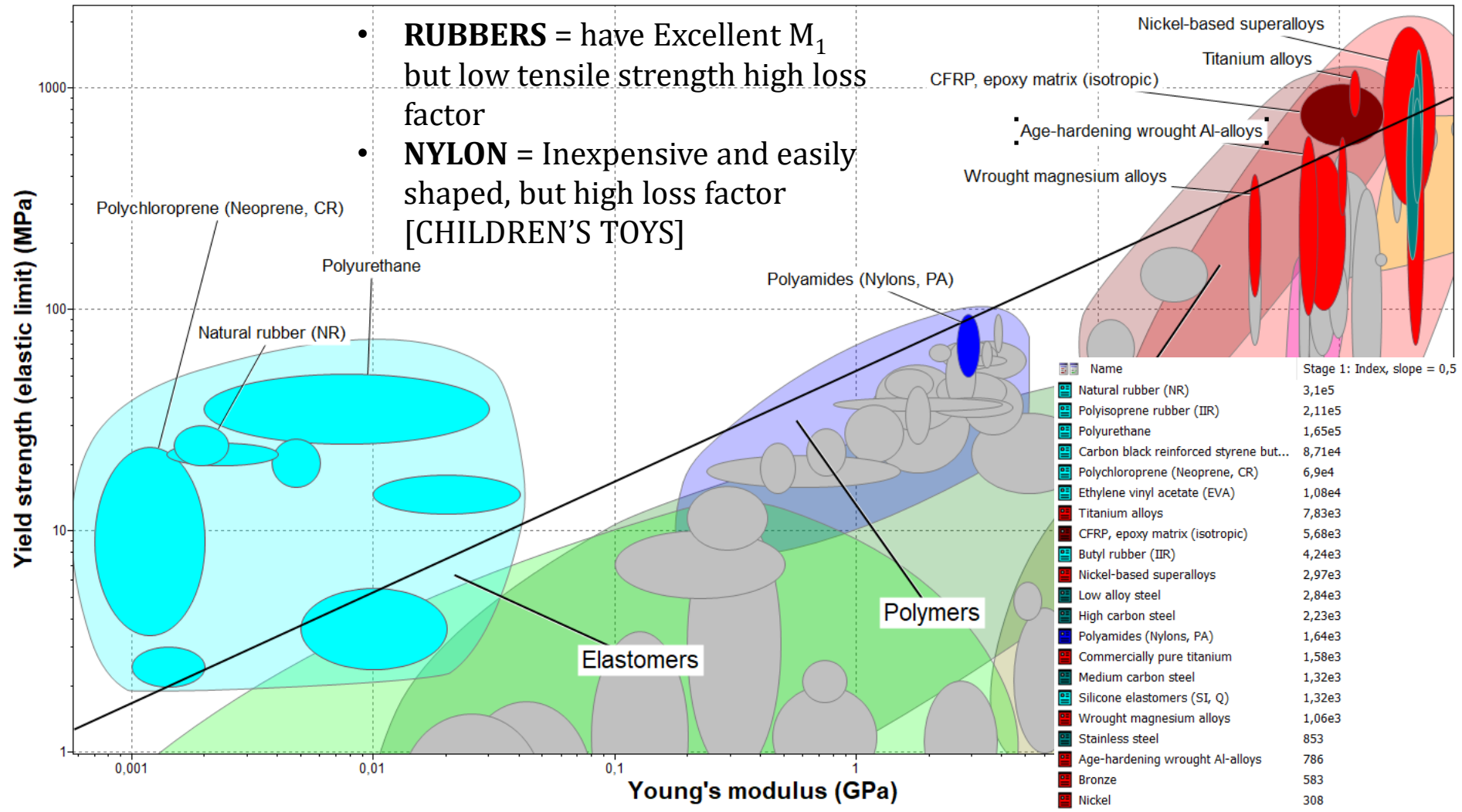
[Solid Mechanics Part I Kelly]





# Case Study 10: Materials for Small Springs

- **CFRP** = Comparable in performance with steel; expensive [TRUCK SPRINGS]
- **STEEL** = The traditional choice: easily formed and heat treated
- **TITANIUM** = Expensive, corrosion resistant





## **Case Study** **Materials for Springs**

$$W_{el} = \frac{\sigma_y^2}{2E}$$

*Valid for axial springs  
Because much of the material is not  
fully loaded*



**PAY ATTENTION**

$$W_{el} = \frac{\sigma_y^2}{3E}$$

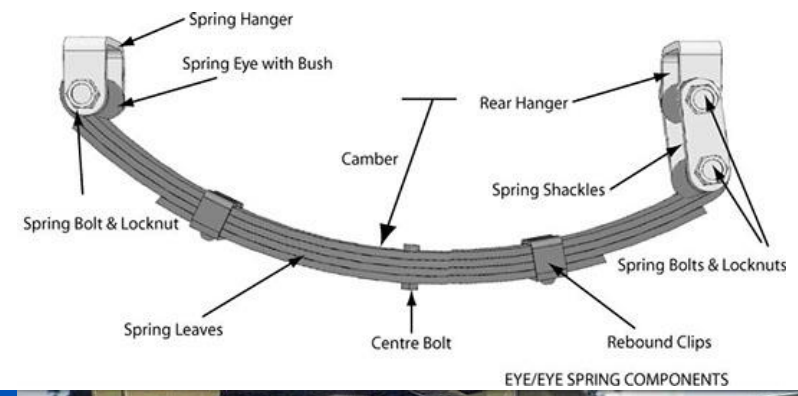
***For torsion springs (less efficient)***





# Case Study Materials for Springs

$$W_{el} = \frac{\sigma_y^2}{4E}$$



*For leaf springs (less efficient)*



**Case Study 9:  
Materials for Light Springs**

$$\left\{ \begin{array}{l} \sigma_y \geq \sigma \\ \varepsilon = \frac{\sigma}{E} \end{array} \right.$$

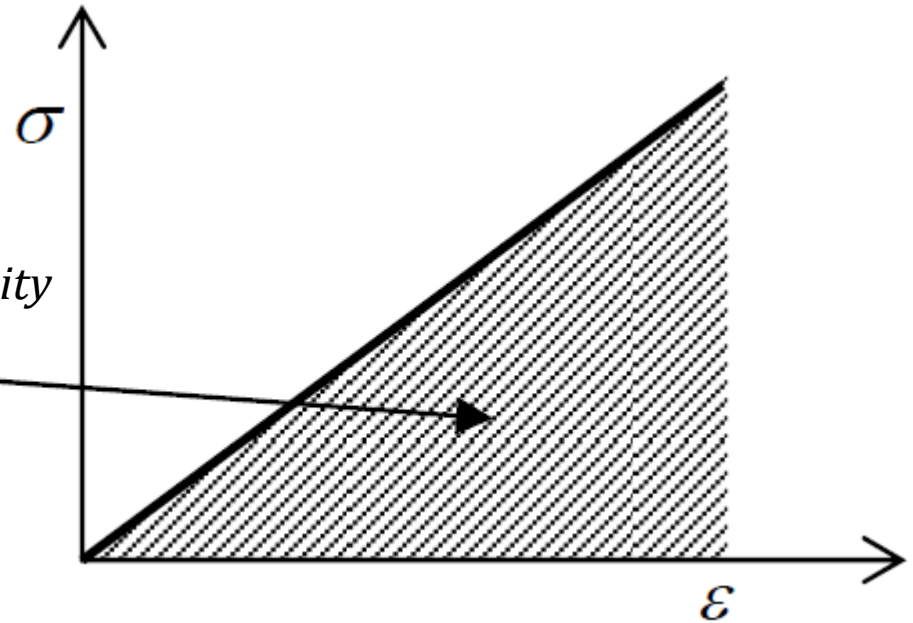
$$dV = dx dy dz$$

$$\left\{ \begin{array}{l} m = V \cdot \rho \\ W_{el} = \frac{1}{2} \int \sigma \cdot \varepsilon dV = \frac{1}{2} \sigma \cdot \varepsilon \cdot V \end{array} \right.$$

*Total strain energy in the piece considered*

*Strain energy density*

$u$



*LIGHT?? →  $\frac{V}{\rho}$  FREE VARIABLE!!*

$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

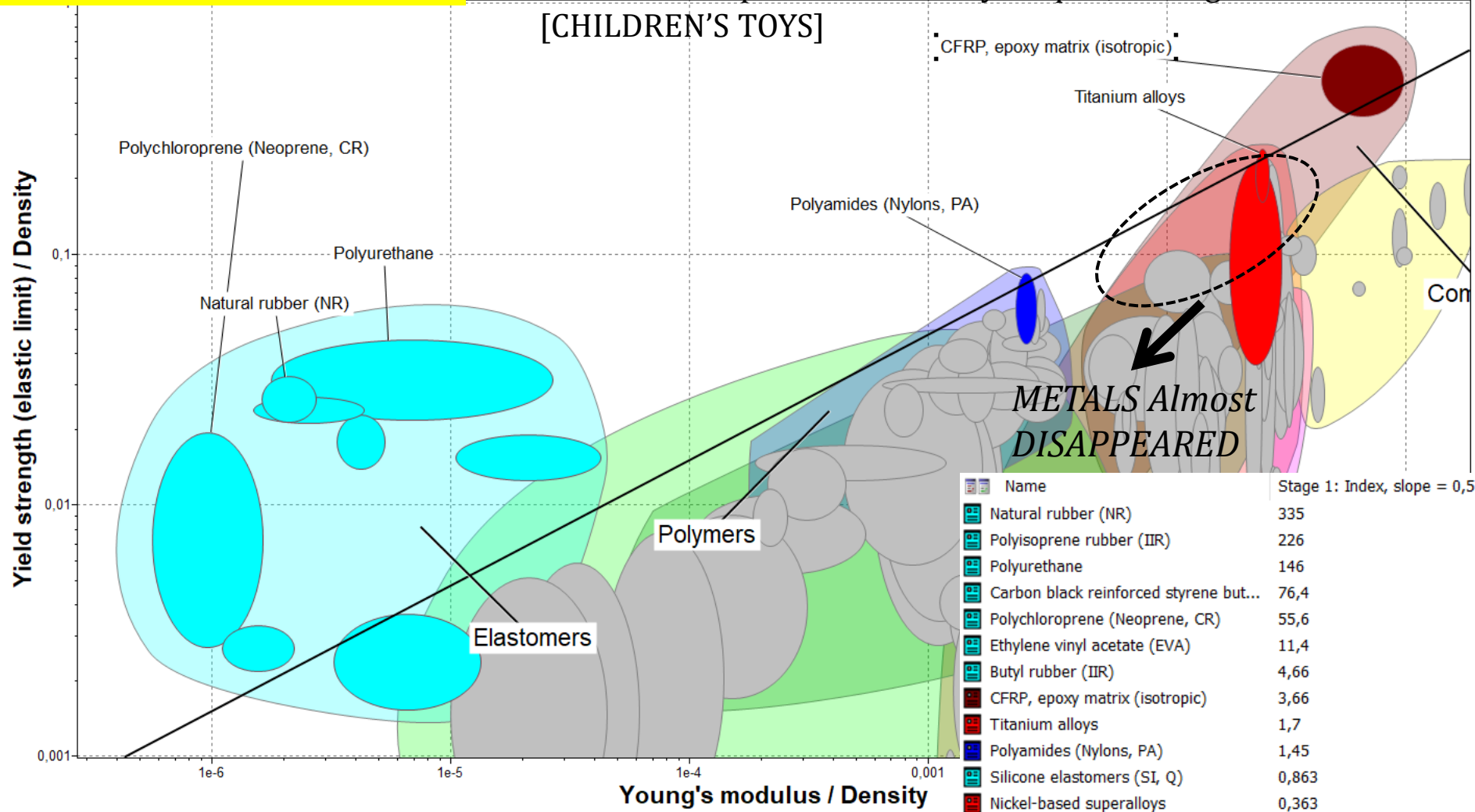
*Total strain energy  
PER UNIT OF MASS*

[Solid Mechanics Part I Kelly]



- **CFRP** = Comparable in performance with steel; expensive [TRUCK SPRINGS]
- **RUBBERS** = 20 times better than Steel; but low tensile strength high loss factor
- **NYLON** = Inexpensive and easily shaped, but high loss factor [CHILDREN'S TOYS]

## Case Study 11: Materials for Light Springs





## Case Study 12: Materials for Car Body

Some context → Car Evolution



1932 Ford Model B



1934 Bonnie and Clyde car



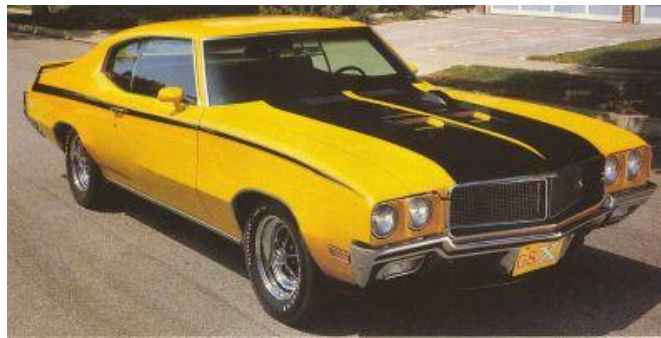


## Case Study 12: Materials for Car Body

Some context → Car Evolution



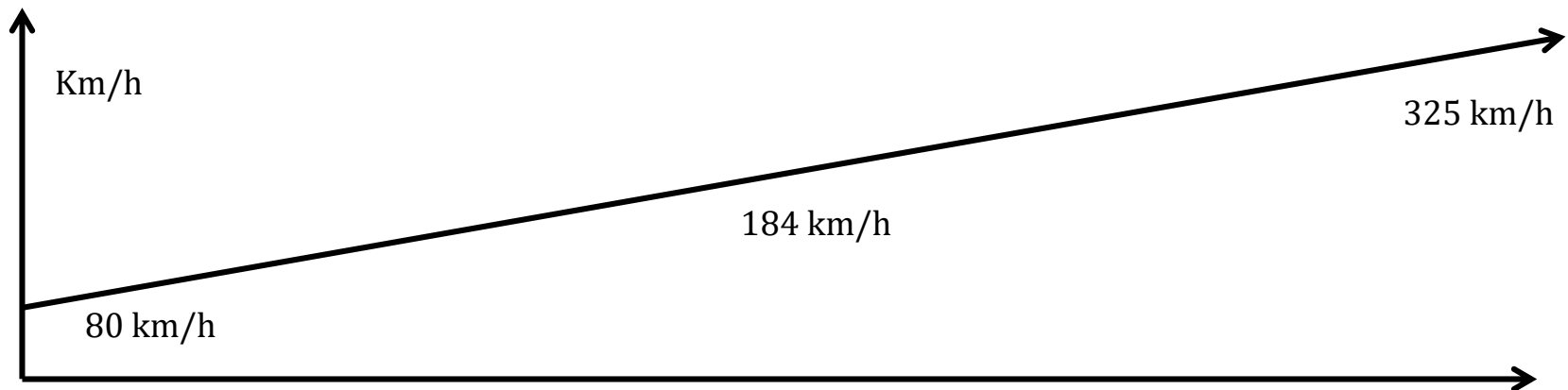
1932 Ford Model B



1970 Buick GSX



2010 Ferrari 458 Italia





## Case Study 12: Materials for Car Body

*Deformation?? → ENERGY CONSUMPTION*



At first, automotive industry  
move to too deformable cars  
and then move to have a mix  
FOR PEOPLE SAFETY



Rover 100 (1997)



Rover 100 (2017)

Sometimes exaggerate





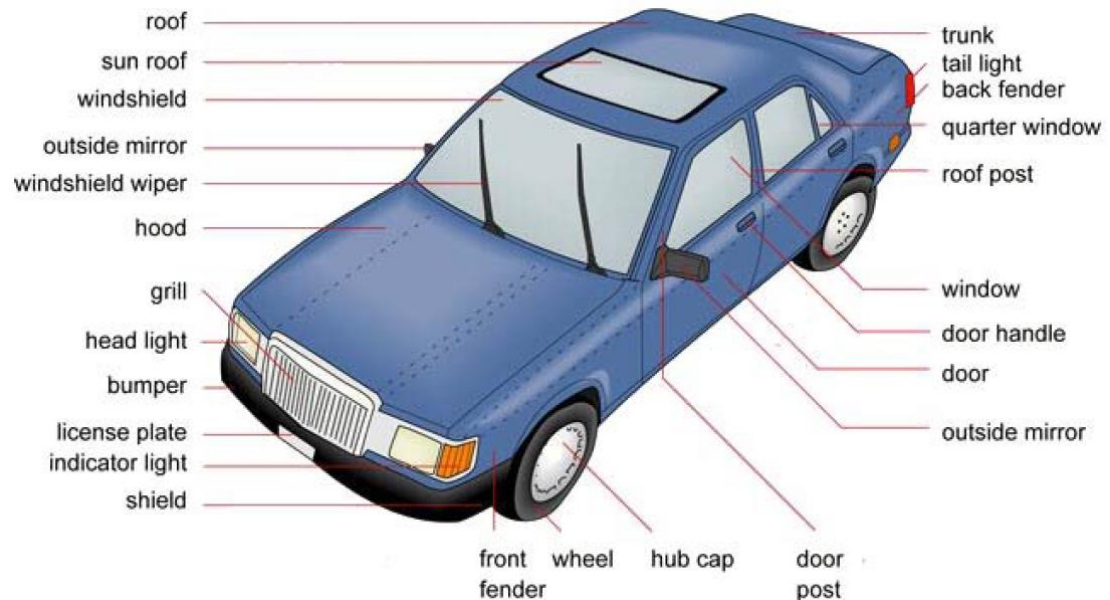
**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**

Objective	<ul style="list-style-type: none"> <li>Maximize plastic deformation at high load</li> </ul>
Constraints	<ul style="list-style-type: none"> <li>Geometry</li> <li>High <math>\sigma_y</math></li> <li>Division for price</li> <li>Consider manufacture</li> </ul>
Free Variables	<ul style="list-style-type: none"> <li>Choice of the material</li> </ul>

LIGHT??  $\rightarrow m$

$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

Total strain energy  
PER UNIT OF MASS





**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**

*LIGHT?? → m*

$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

*Total strain energy  
PER UNIT OF MASS*

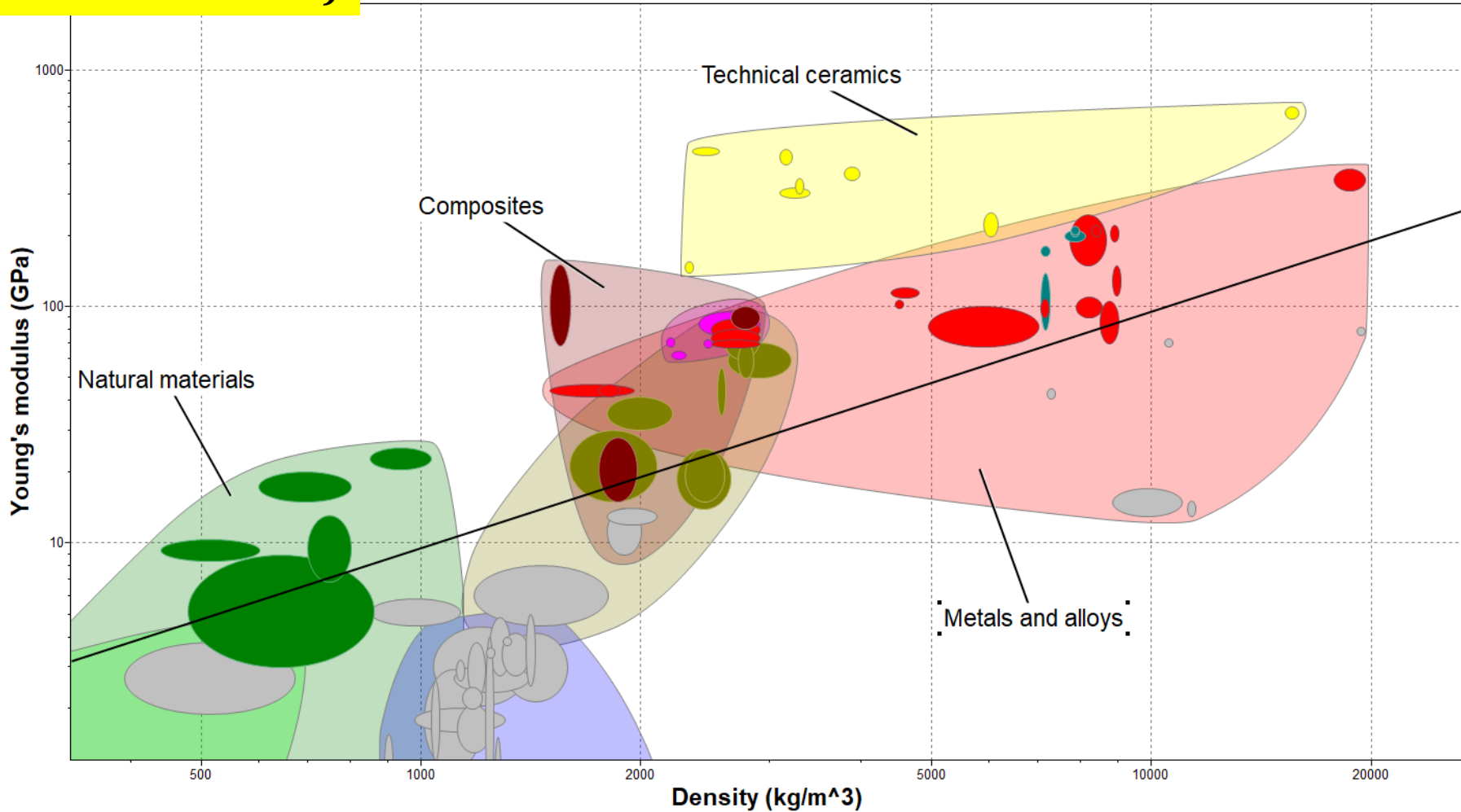
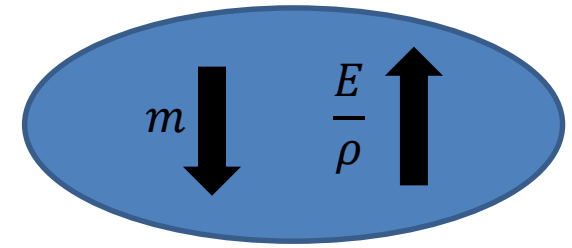
Objective	<ul style="list-style-type: none"><li>• Maximize plastic deformation at high load</li></ul>
Constraints	<ul style="list-style-type: none"><li>• <i>Geometry</i></li><li>• <i>High <math>\sigma_y</math></i></li><li>• Division for price</li><li>• Consider manufacture</li></ul>
Free Variables	<ul style="list-style-type: none"><li>• Choice of the material</li></ul>

*Steps:*

- *Stiffness selection (Take off flexible materials)*
- *Yield strength selection to minimize the costs (Automotive)*
- *Minimum Yield Strength*
- *Maximization of stored energy*

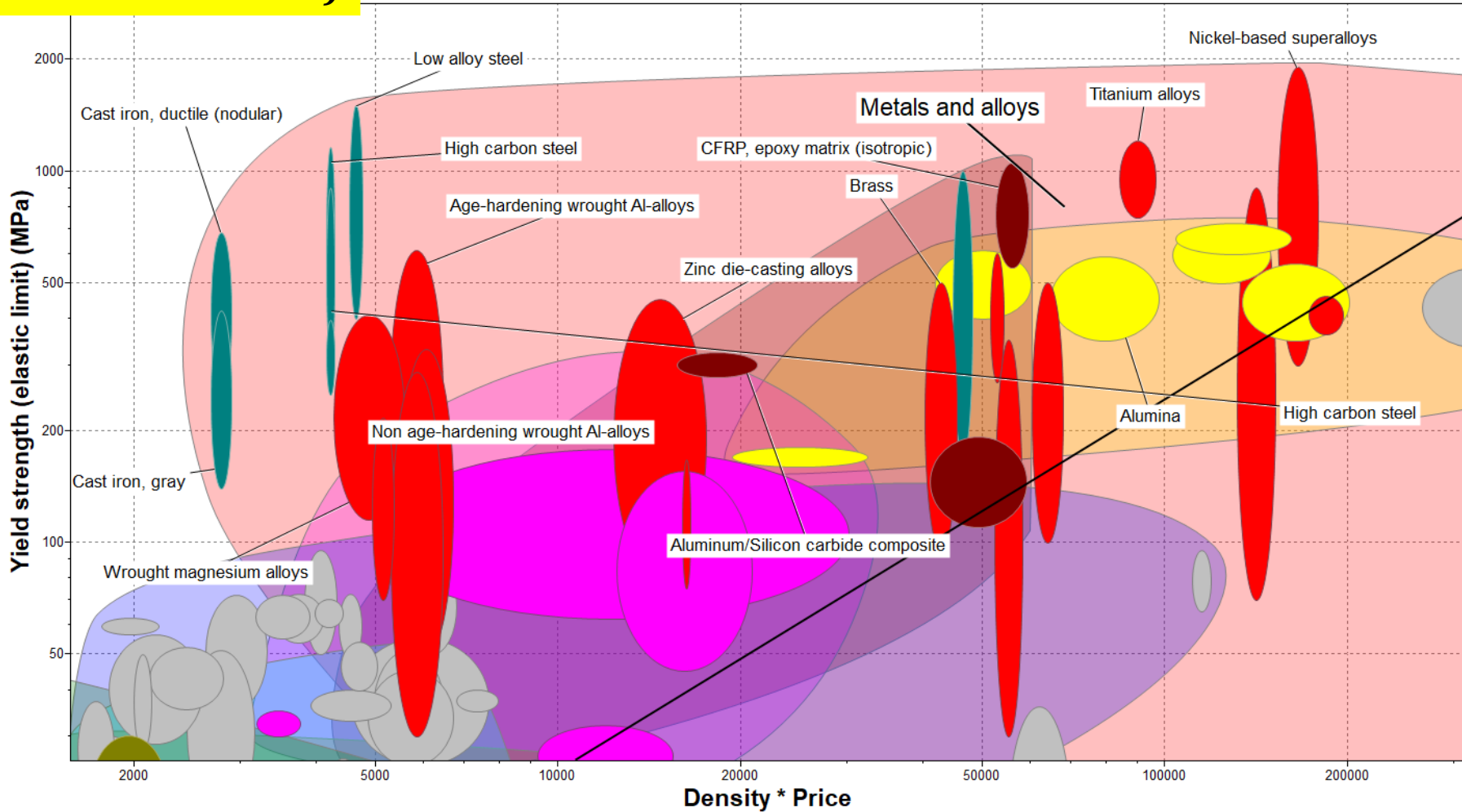


**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**





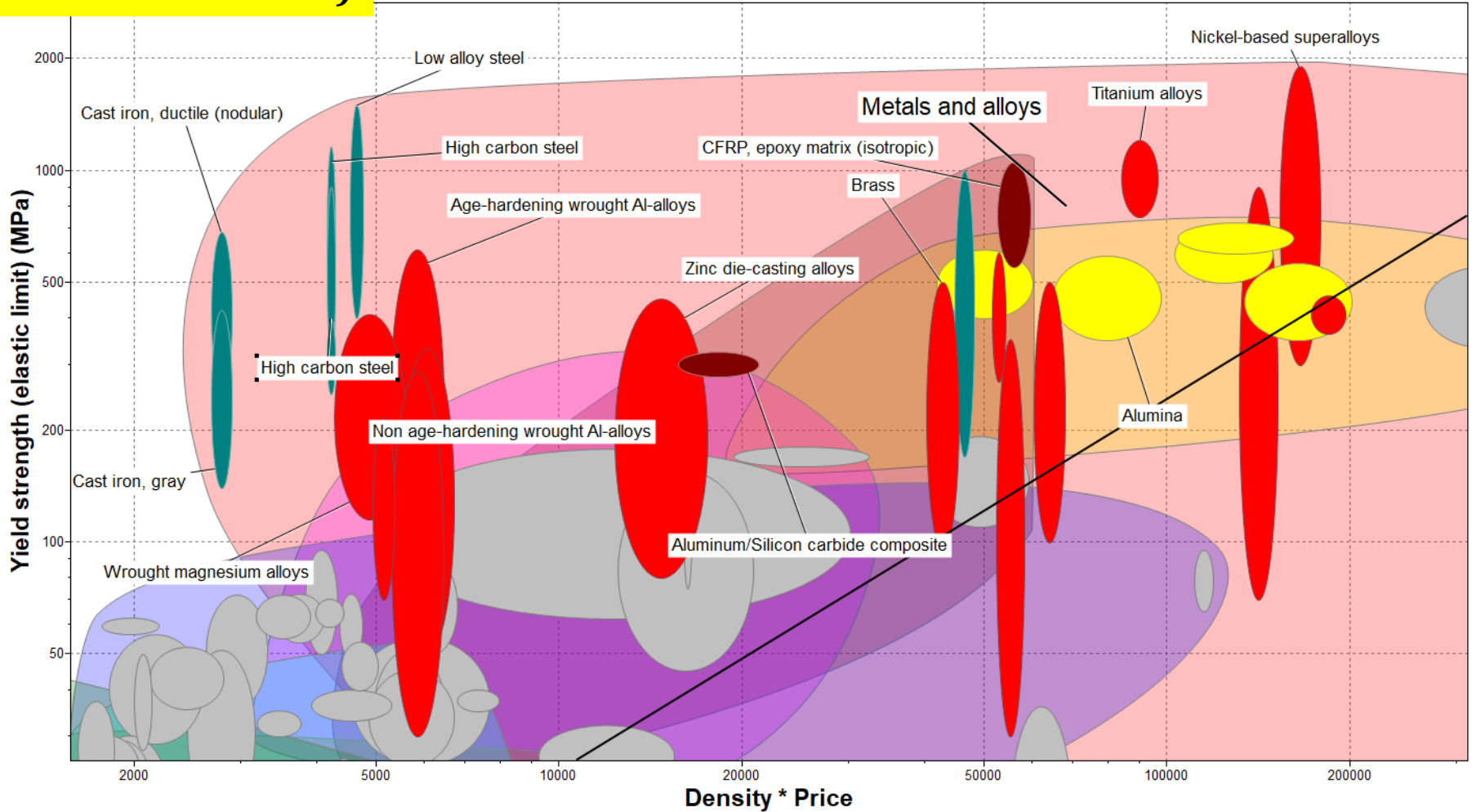
# Case Study 12: Materials for Car Body (Car Hood or Car Door)





# Case Study 12: Materials for Car Body (Car Hood or Car Door)

+ minimum  $\sigma_y$  (200 MPa)

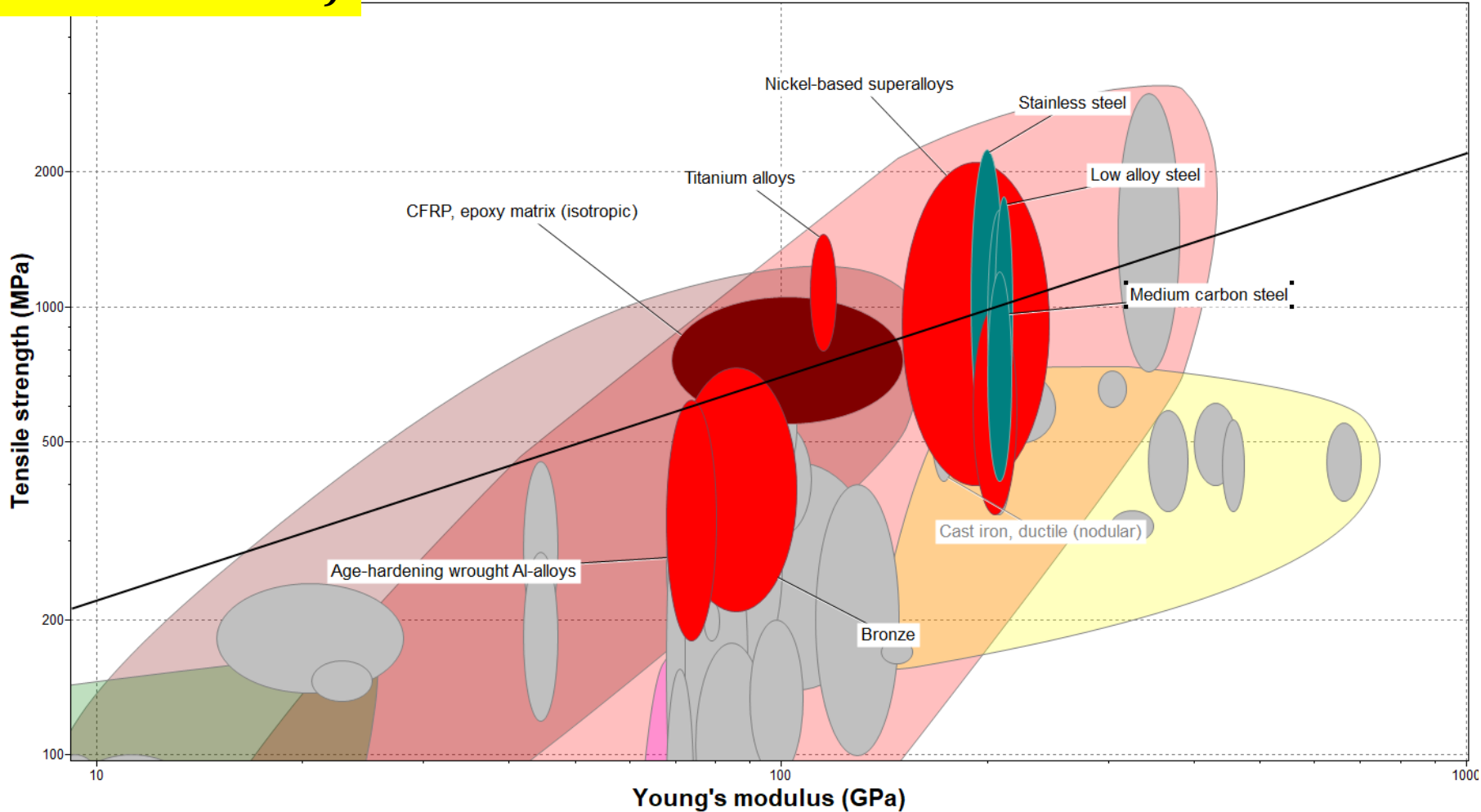




**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**

$$W_{el} = \frac{\sigma_f^2}{2E} = M_1$$

*Total strain energy  
PER UNIT OF VOLUME*

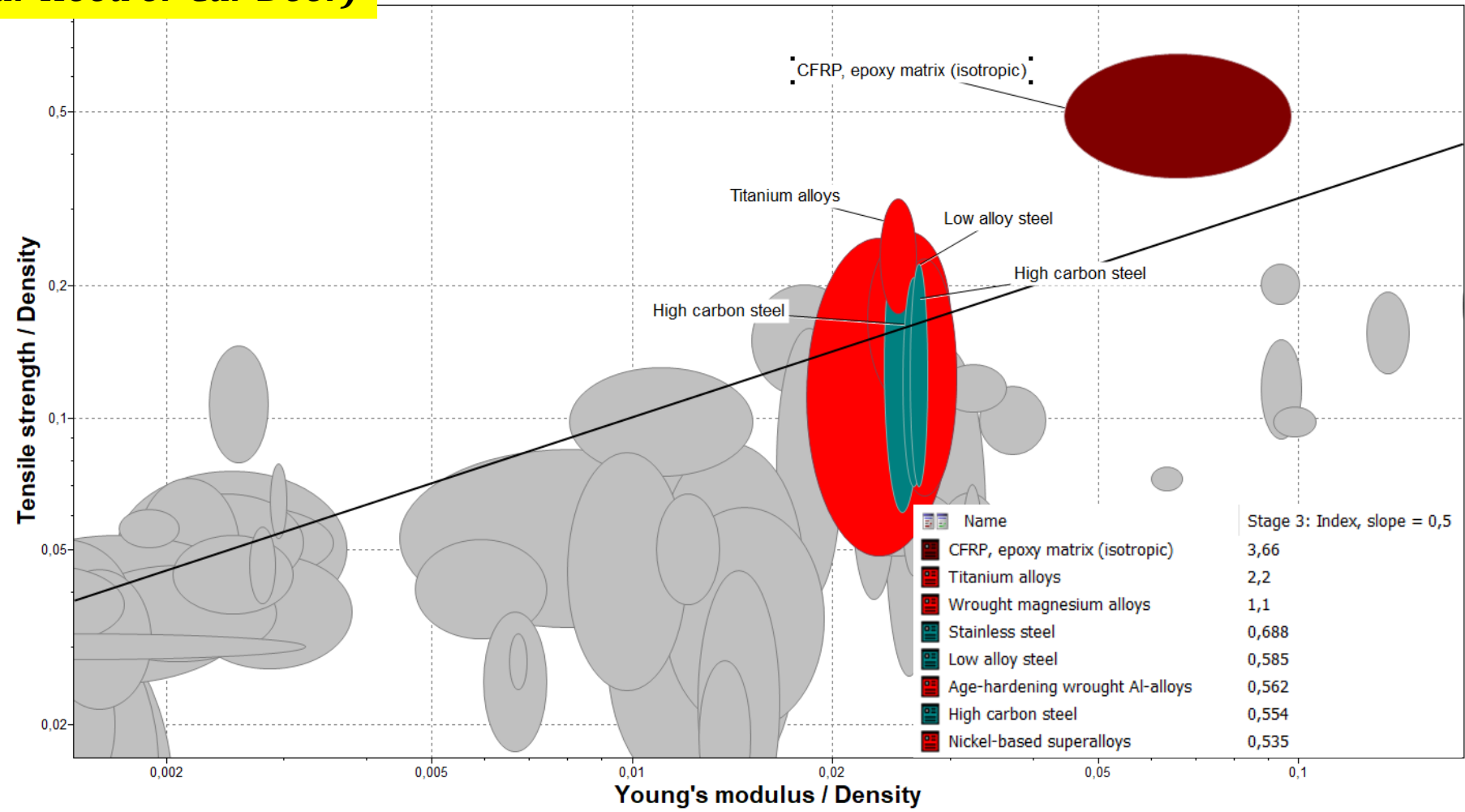




$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**

*Total strain energy  
PER UNIT OF MASS*





$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

Total strain energy  
PER UNIT OF MASS

**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**



*Lamborghini Huracan 2015*

Name	Stage 3: Index, slope = 0,5
CFRP, epoxy matrix (isotropic)	3,66
Titanium alloys	2,2
Wrought magnesium alloys	1,1
Stainless steel	0,688
Low alloy steel	0,585
Age-hardening wrought Al-alloys	0,562
High carbon steel	0,554
Nickel-based superalloys	0,535





## **Case Study 12: Materials for Car Body (Car Hood or Car Door)**

# *Manufacturing Consideration*

### Changing the carbon fiber manufacturing process

Lamborghini's innovation is a product and a process called Forged Composite. This material starts off as a sheet of uncured plastic that is mixed with short lengths of randomly placed carbon fiber strands. Unlike traditional pre-preg carbon fiber cloth, you don't have to carefully cut this material and lay it out precisely in a mold. You just have to cut off the right mass and put the chunk into a hot press mold. You squeeze it, heat it and you're done. The part that comes out of the mold is as light (or lighter) and as stiff (or stiffer) than a conventionally laid-up carbon fiber part, and you can produce it in minutes rather than hours.

You can now treat carbon fiber the way the automobile industry has treated steel, aluminum, and unreinforced plastic for decades.

This changes the rules of manufacturing because you can now treat carbon fiber the way the automobile industry (and every other manufacturing industry) has treated steel, aluminum, and unreinforced plastic for decades: You just stamp out the parts you need. As automakers look to the future of increased CAFE standards and lighter-weight vehicles, making parts out of carbon fiber without the extra labor expense is a killer app.

"By continuing to develop our patented forged composite materials, we are able to create a product that can enhance Lamborghini super sports cars in both their performance and their appearance," said Maurizio Reggiani, Director of R&D for Lamborghini. "The ability to leverage this kind of lightweight material gives Lamborghini an advantage that will benefit our cars – as well as production process – in the future."

*[[www.digitaltrends.com/cars/lamborghini-forged-carbon-fiber-manufacturing-process](http://www.digitaltrends.com/cars/lamborghini-forged-carbon-fiber-manufacturing-process)]*



# Vacuum and pressure bag molding

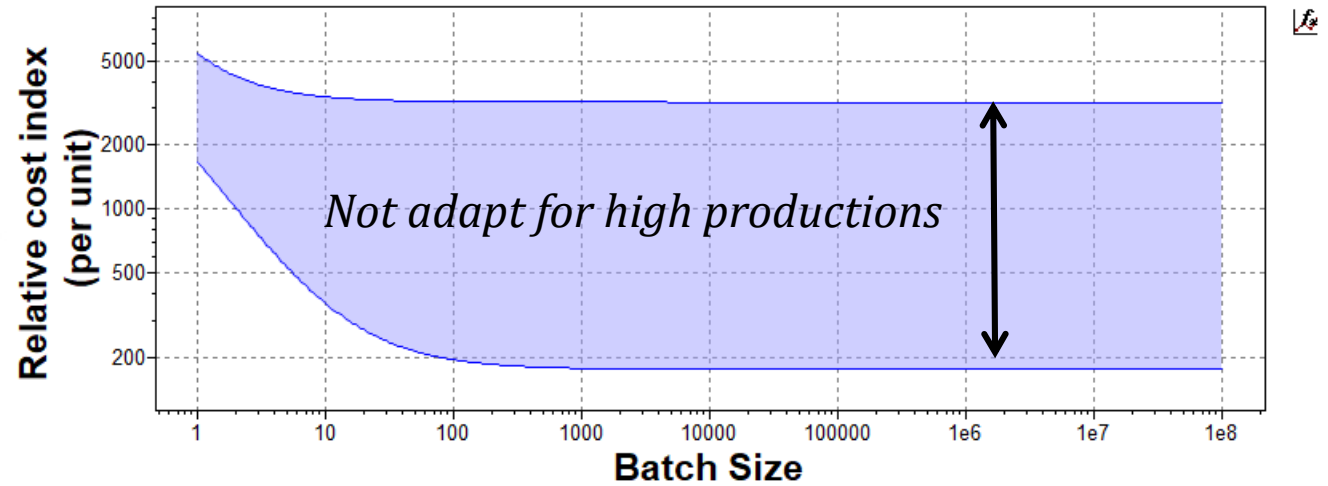
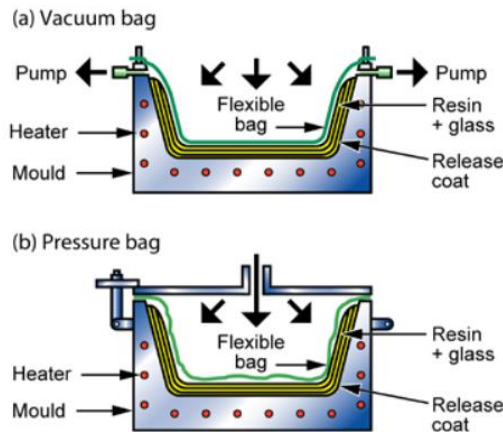
## Case Study 12: Materials for Car Body (Car Hood or Car Door)

### Cost model and defaults

Relative cost index (per unit)

178 - 3,2e3

Parameters: Material Cost = 7,33EUR/kg, Component Mass = 1kg, Batch Size = 1e3, Overhead Rate = 137EUR/hr, Discount Rate = 5%, Capital Write-off Time = 5yrs, Load Factor = 0,5



Material Cost=7,33EUR/kg, Component Mass=1kg, Overhead Rate=137EUR/hr, Capital Write-off Time=5yrs, Load Factor=0,5, Discount Rate=5%

Capital cost		3,01e4	-	7,52e5	EUR
Material utilization fraction		0,85	-	0,95	
Production rate (units)	<i>No way for a commercial car</i>	0,05	-	1	/hr
Tooling cost		752	-	3,01e3	EUR
Tool life (units)		100	-	1e3	

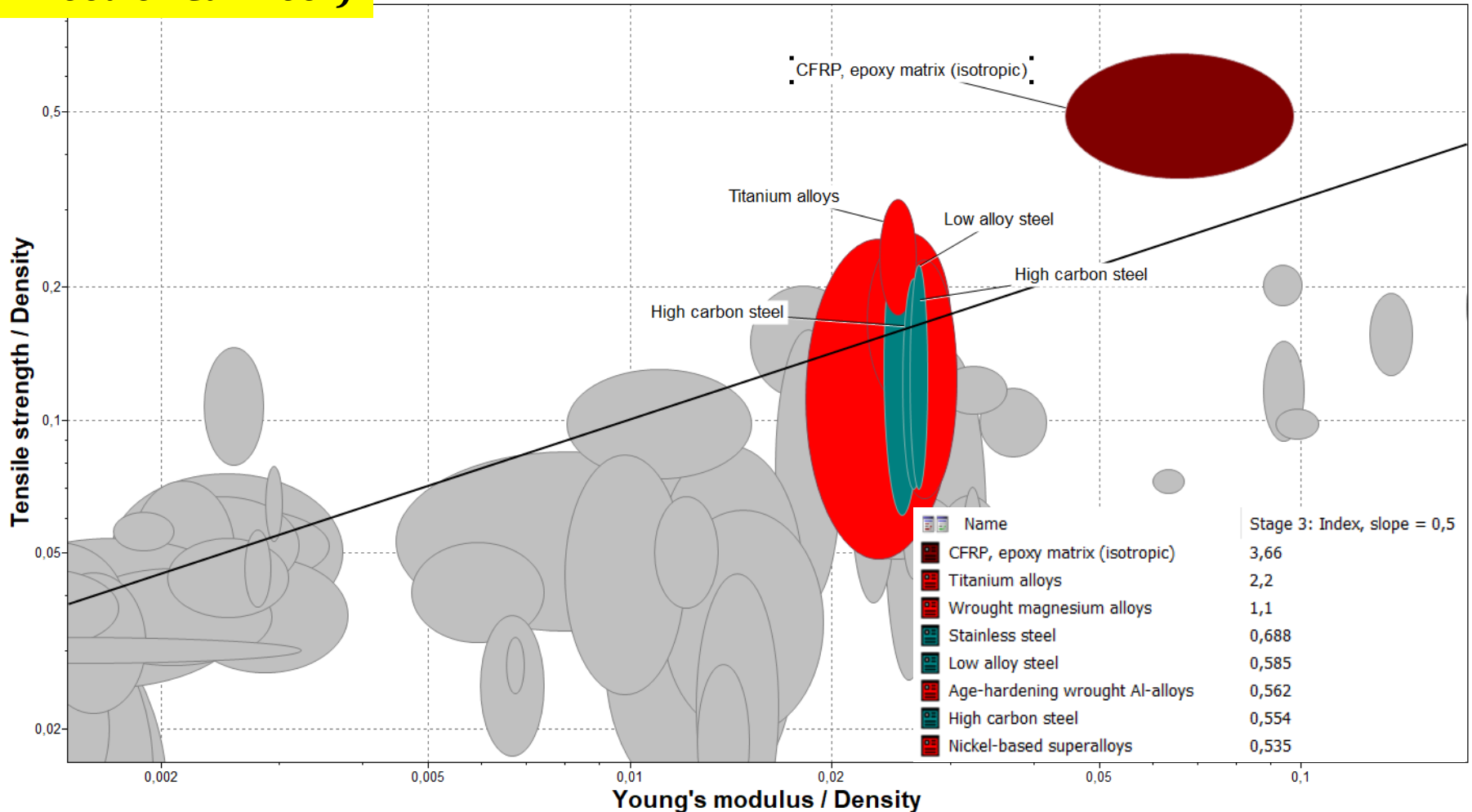


$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

Total strain energy  
PER UNIT OF MASS

**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**

# And which Metal?





# Stamping

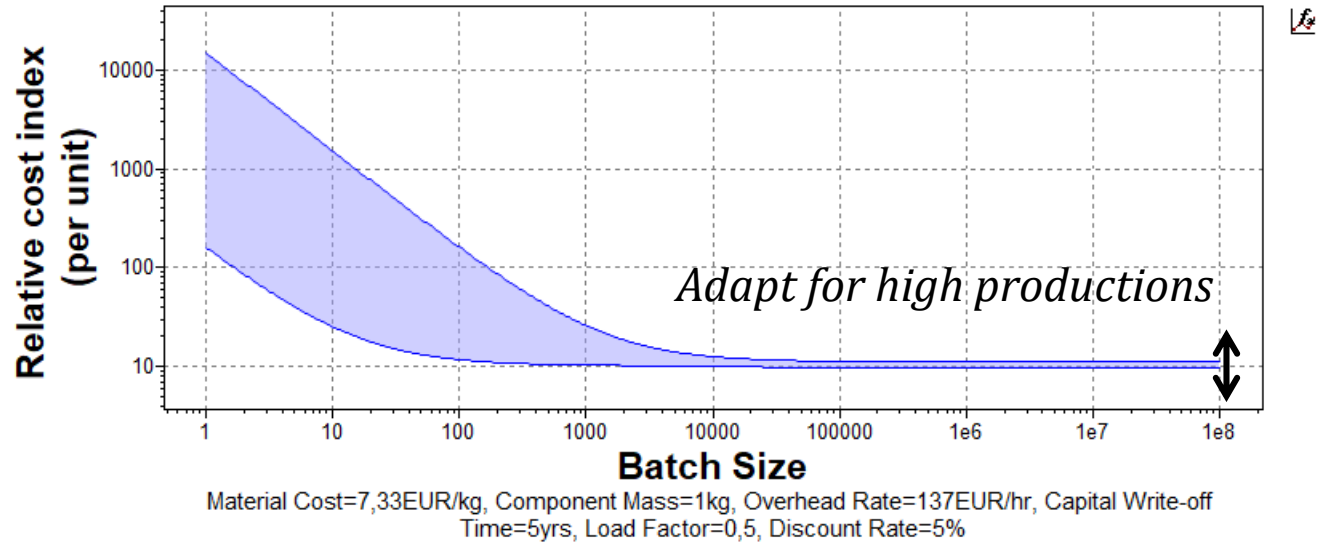
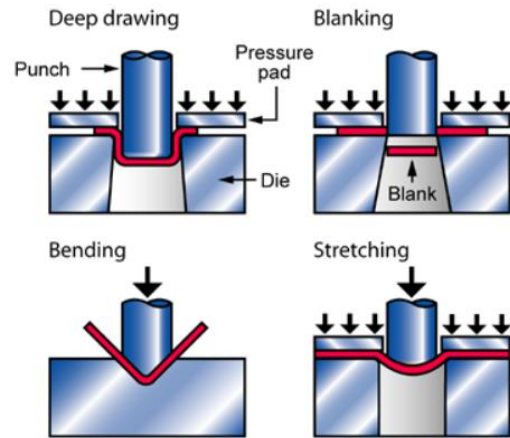
## Case Study 12: Materials for Car Body (Car Hood or Car Door)

### Cost model and defaults

Relative cost index (per unit)

**i** 10,3 - 26

Parameters: Material Cost = 7,33EUR/kg, Component Mass = 1kg, Batch Size = 1e3, Overhead Rate = 137EUR/hr, Discount Rate = 5%, Capital Write-off Time = 5yrs, Load Factor = 0,5



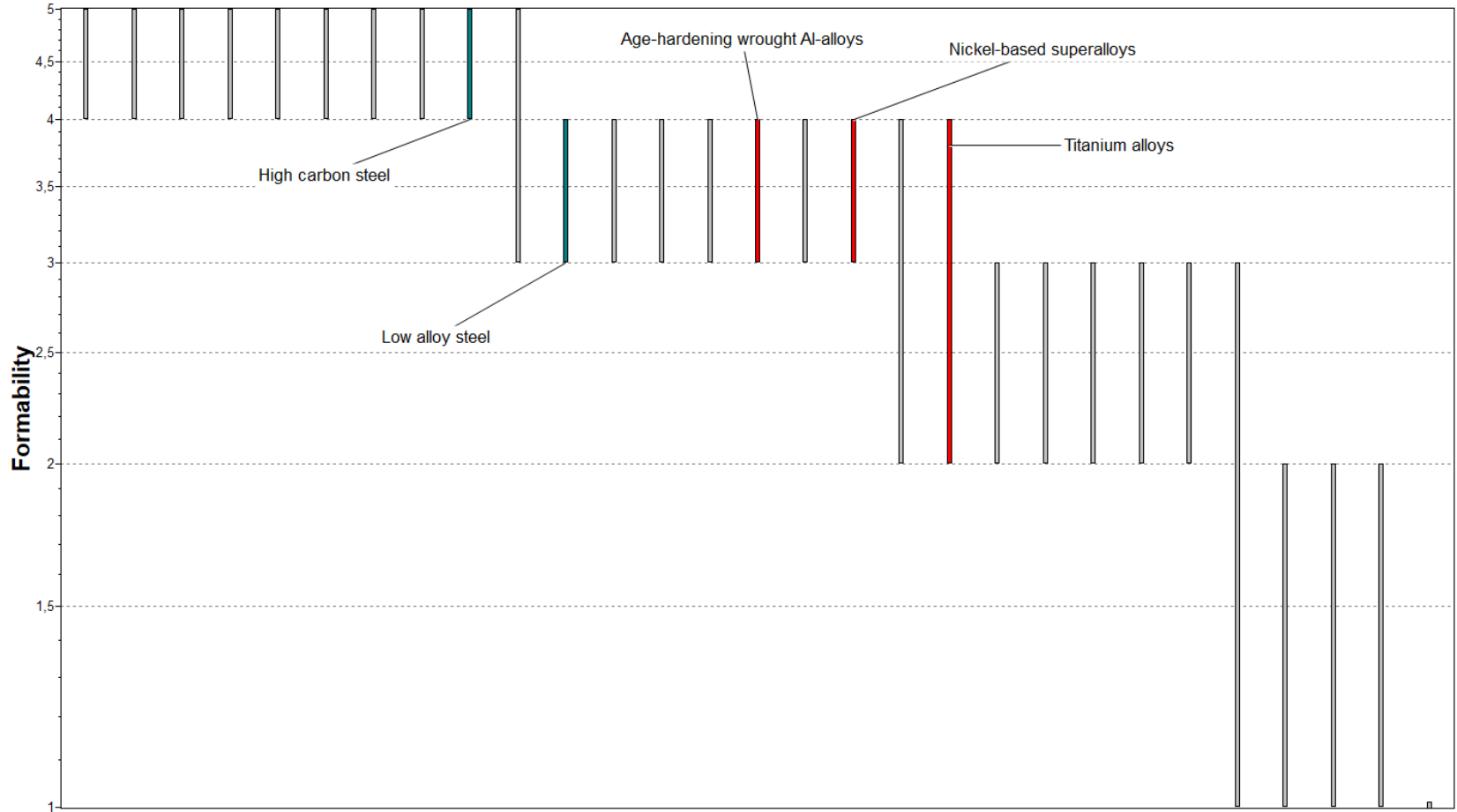
Capital cost	<b>i</b>	7,52e3	-	7,52e4	EUR
Material utilization fraction	<b>i</b>	0,7	-	0,8	
Production rate (units)	<b>i</b>	200	-	5e3	/hr
Tooling cost	<b>i</b>	150	-	1,5e4	EUR
Tool life (units)	<b>i</b>	1e4	-	1e6	



# PROCESSABILITY

## Case Study 12: Materials for Car Body (Car Hood or Car Door)

*Metals easy to stamp*



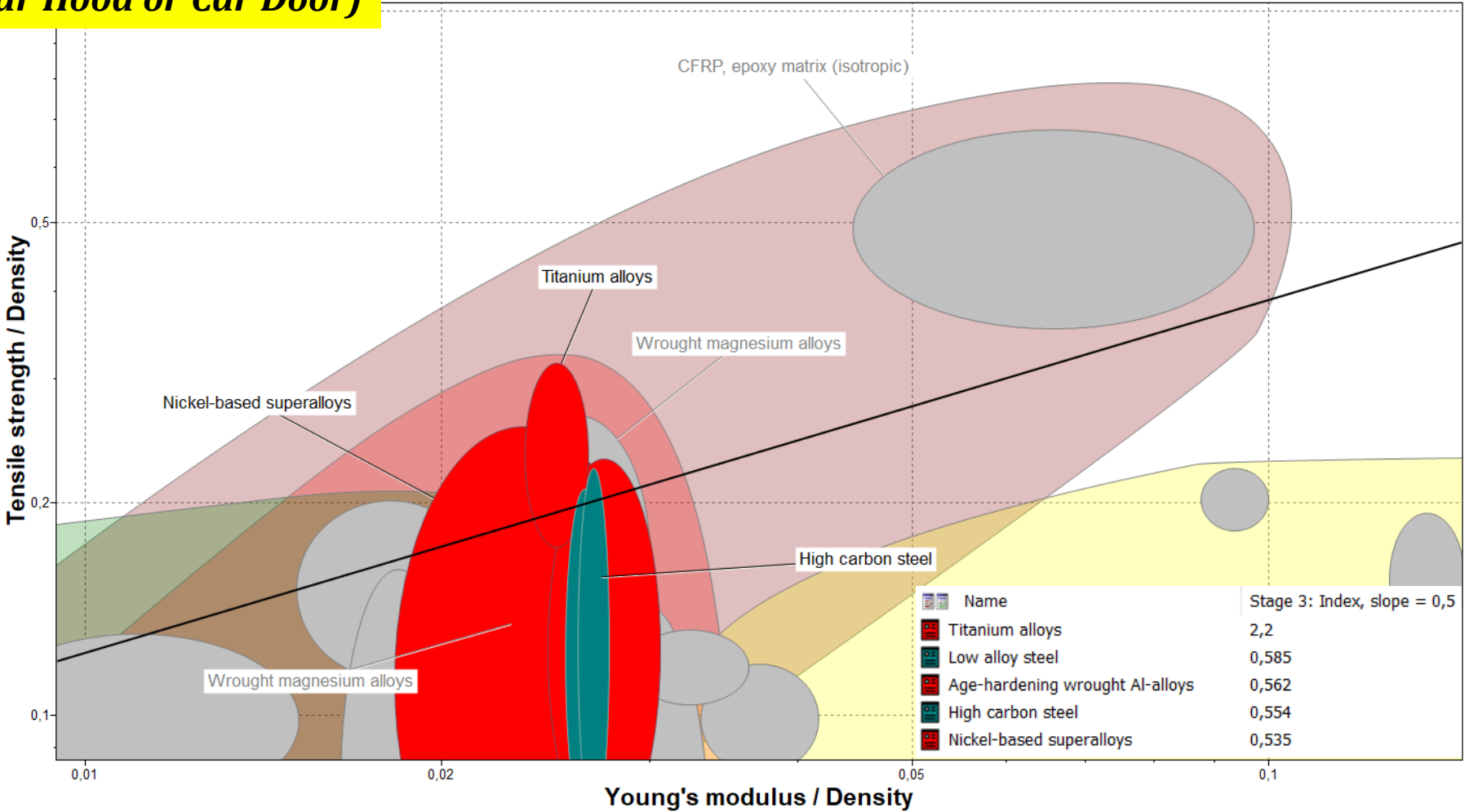


$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

+ Minimum Formability (4)

Total strain energy  
PER UNIT OF MASS

**Case Study 12:  
Materials for Car Body  
(Car Hood or Car Door)**

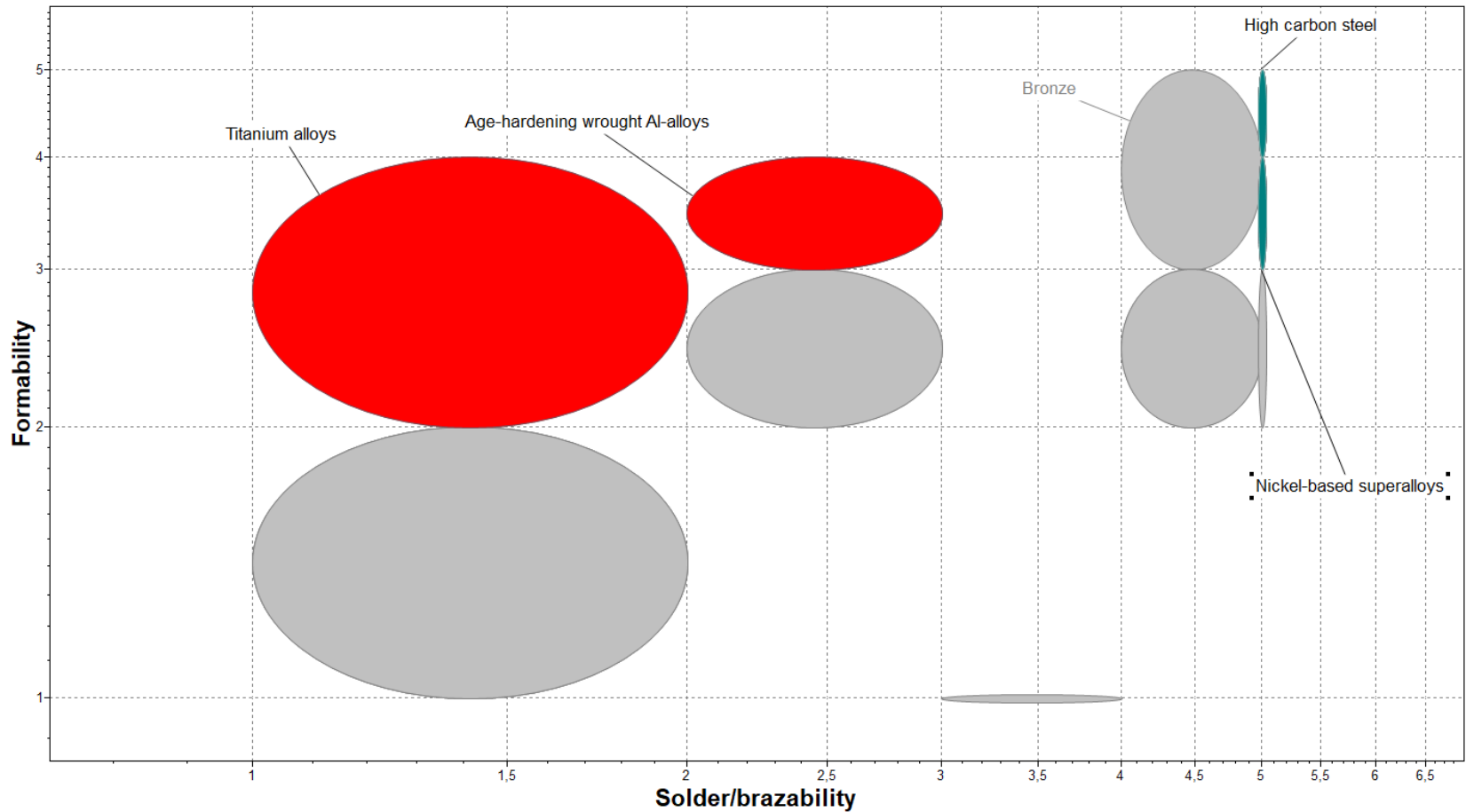




# PROCESSABILITY

## Case Study 12: Materials for Car Body (Car Hood or Car Door)

Metals easy to stamp



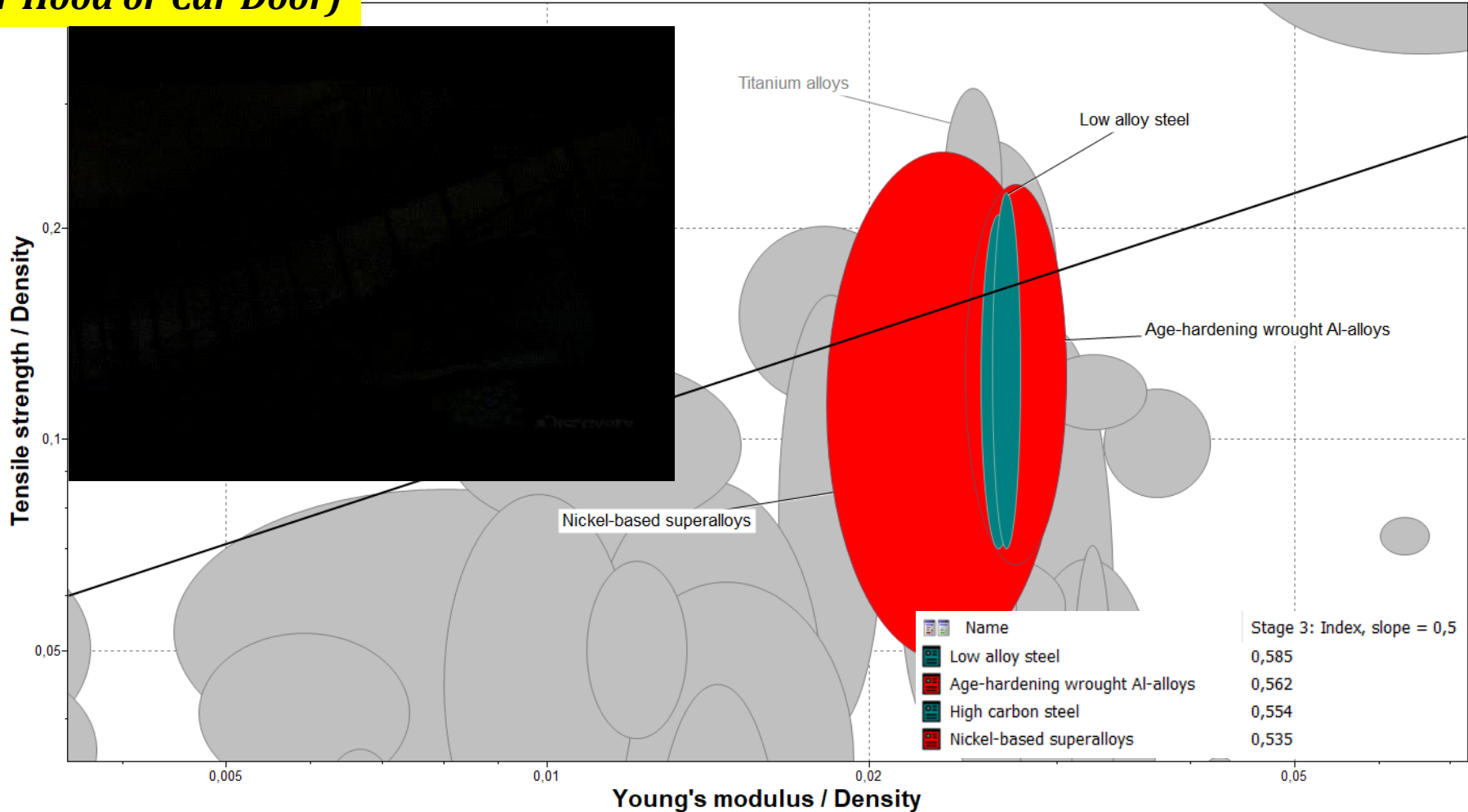


$$\frac{W_{el}}{\rho} = \frac{\sigma_f^2}{2 \cdot E \cdot \rho} = M_2$$

Total strain energy  
PER UNIT OF MASS

- + Minimum Formability (4)
- + Minimum Brazability (3)

**Case Study 12:**  
**Materials for Car Body**  
**(Car Hood or Car Door)**







## Case Study 12: Materials for Car Body (Car Hood or Car Door)



*BMW M3 – Low alloy steel*



*Audi A8 – Al-alloys*

Name	Stage 3: Index, slope = 0,5
Low alloy steel	0,585
Age-hardening wrought Al-alloys	0,562
High carbon steel	0,554
Nickel-based superalloys	0,535

**Deeper selection?**



**LEVEL 3**

[<https://www.cartalk.com/blogs/jim-motavalli/steel-vs-aluminum-lightweight-wars-heat>]



# Materials Selection Steps

“DON'T  
BELIEVE  
EVERYTHING  
YOU READ  
ON THE  
INTERNET”

~ ABRAHAM  
LINCOLN



Decide Design  
Requirements

Get rid of Candidates that don't  
fit constraints e.g. max service  
temperature isn't high enough

Optimize on Objectives  
e.g. low mass, low cost,  
high strength

Scrutinize candidate  
shortlist – do I have valid  
properties

- Materials Selection is about trade-offs, not one right answer
- Environmental legislation, processability and the security of the supply chain are important factors, along side mechanical and thermal performance

Optional:

Decide on strategy to fill  
knowledge gaps